A CONVENIENT METHOD OF CORRELATING RHEOLOGY DATA

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A convenient method of correlating rheology data is described. The method is based on a spreadsheet plot of a rheological parameter (for example yield stress) against $Cw^{\mathbf{n}}$ where Cw is solids concentration by weight and **n** is an input parameter. An exponential trend line is fitted to the Yield Stress versus $Cw^{\mathbf{n}}$ plot and **n** is manually varied until the highest R^2 fit is obtained. Having obtained the exponential trend equation, the yield stress is plotted against Cw with the correlating curve based on an exponential function of $Cw^{\mathbf{n}}$ included. The method is simple and quick and consistently gives a superior fit to the data than more conventional methods. It provides a smooth equation allowing parameter values to be obtained for any concentration within the concentration range tested and provides a viable means of extrapolating to higher and lower concentrations. Description of the method is confined to the Bingham model although the method proposed is equally applicable to other rheological models. The paper illustrates the method by applying it to rheology data for a range of slurries of differing solids SGs and particle size. The relationship to conventional correlation methods is discussed.

KEY WORDS: Rheology, Slurry, Viscometer, Bingham Plastic

1. BACKGROUND

By way of background information, in most industrial applications a slurry cannot usually be characterised as either non-settling or settling. The fine particle fraction (say, less than about 40µm) and the liquid may form a nearly homogeneous “carrier fluid” or “vehicle” slurry with non-settling behaviour. Slurries in mineral processing, for example wet tailings, typically have average particles sizes of 20 to 100µm with maximum sizes of up to 500 to 1000µm. Heavy ore concentrates for long distance pipeline pumping may have about 85% less than 40 µm with maximum sizes about 100 µm. The corresponding slurries can be termed complex because they cover an intermediate area between homogeneously and heterogeneously flows or ”non-settling” and “settling” types of mixtures, respectively.

Numerous theories have been developed over the years to predict pipeline transport behaviour of settling particles in a fluid. These theories are based on the viscosity of the fluid, not the viscosity of the total mixture, which would include a mechanical interference component due to the particles. But when viscometer tests are performed on a typical intermediate type slurry it is the total viscosity which is measured. A.D. Thomas (2010) considers that the resulting measured viscosity, μ_{tot}, is made up of an inherent viscosity component, μ_{inh}, and a mechanical interference component, μ_{mi}, i.e. $\mu_{tot}=\mu_{inh}$ μ_{mi}. To properly apply the settling slurry theories the inherent viscosity should be used in place of the fluid viscosity and a means of separating the mechanical component of the measured viscosity from the inherent viscosity of the finer particles is required. A.D. Thomas (2010) proposed such a method after analysing test results in terms of Bingham model parameters for over 200 slurries of the above mentioned intermediate type based on viscometric measurements with a rotational cylindrical viscometer.

2. INTRODUCTION TO CURRENT PAPER

The objective of the current paper is to present a convenient method of correlating rheological data in terms of the Bingham yield stress and plastic viscosity versus solids content. These parameters form the basis also for turbulent pipeline design for friction loss calculations and velocity requirement and for the aforementioned determination of total and inherent viscosities. Rheology data obtained from viscometer tests on a slurry are typically generated as flow curves of shear stress versus shear rate for a range of solids concentrations. To facilitate pipe flow predictions, a rheological model such as the Bingham Plastic model is then fitted to the shear stress-shear rate curves. Figure 1 shows typical shear stress versus shear rate curves obtained in a rotational viscometer.

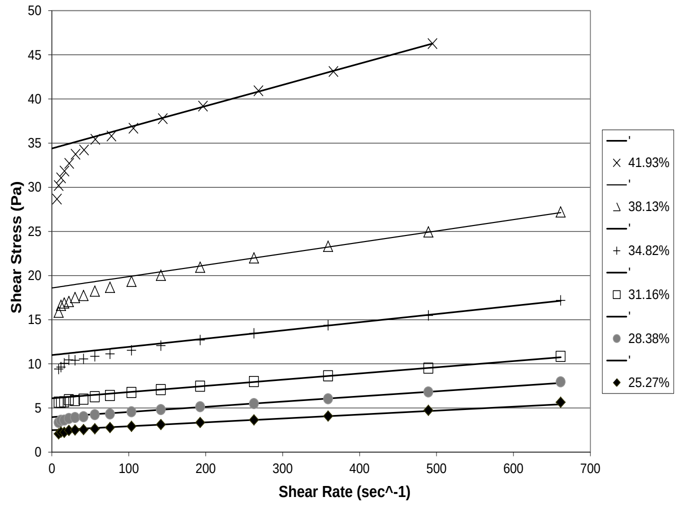


Fig. 1 Coal tailings – Shear stress versus shear rate. Concentration by weight as shown.

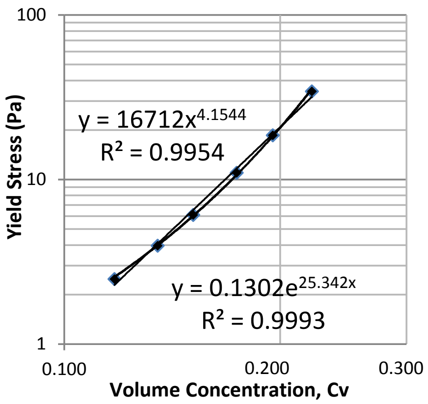
The data in Figure 1 apply to a minus 850 micron coal tailings slurry (p80 = 48 microns, d50 = 5 microns, solids SG 2.54). Straight lines fitted to the high shear rate data define the Bingham parameters. The Bingham yield stress is given by the line intercept on the Shear Stress axis and the slope of the line represents the plastic viscosity. Wilson and Thomas (2008) showed that basing turbulent flow predictions on the straight line Bingham curve and neglecting the curved portion of the data curve at low shear rates makes negligible difference to the turbulent flow prediction.

Having determined the Bingham parameters it is convenient to plot the Bingham Plastic parameters, Yield Stress and Plastic Viscosity, against solids concentration and correlate the data, preferably describing the variation by a mathematical equation to allow the parameters to be obtained for any concentration. Options available for correlating the Bingham parameters with concentration are discussed in the following sections.

2. CONVENTIONAL CORRELATION OF YIELD STRESS

In seeking a correlation it is common to plot the yield stress against volume concentration (Cv). D.G. Thomas (1962) found that for aqueous suspensions of thorium oxide, kaolin, titanium oxide, aluminium oxide, graphite, magnesium oxide, and uranium dioxide having particle sizes in the range 0.35 to 13 microns, the yield stress was proportional to Cv^3. The current authors have similarly found that the yield stress of slurries composed of colloidal sized particles generally varies with Cv to a power around 3.

A.D. Thomas (1999) tested slurries of varying particle size and found that the exponent of Cv increased from around 3 for very fine particle slurries to 7 or 8 or higher as the particle size coarsened. He also found that the data generally followed the Power Law model only approximately and usually followed a more concave path rather than a straight line on log-log co-ordinates, with the concavity increasing as the particle size coarsens. Figure 2 shows the yield stress of the coal tailings of Figure 1 plotted against Cv on logarithmic co-ordinates. The data are described reasonably well (R^2 = 0.9954) by a Power Law equation with an exponent of 4.15. The exponent 4.15 is consistent with the particle size of the coal tailings, which although comprising about half colloidal sized particles, does contain significant coarser particles, and so the exponent can be expected to be higher than the 3 determined by D.G. Thomas for much finer slurries. Note also that the trend of the data in Figure 2 indicates a slightly more concave shape than the straight line Power Law trend line as noted by A.D. Thomas (1999). An exponential trend line, shown dashed, describes this concave shape and gives a better fit to the data (R^2 = 0.9993).



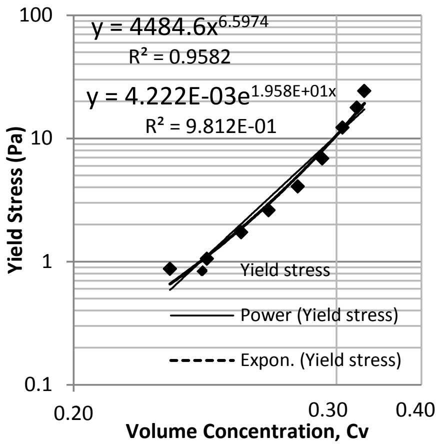


Fig. 2 Yield stress coal tailings versus Cv   Fig. 3 Yield stress copper tails versus Cv

For the coal tailings of Figure 2 the fit of either the Power Law or the exponential correlation can be considered adequate, as indicated by the trend equations. But the yield stress versus Cv plot does not always provide such a good fit. Figure 3 shows a similar plot for a minus 425 micron copper tailings, (p80 = 225 microns, d50 = 85 microns, solids SG 2.71). The data follow a pronounced concave path which deviates from the Power Law correlation significantly for the highest and lowest yield stress values. Note that the Power Law exponent is 6.60 which is higher than the 4.15 exponent of the coal tailings in keeping with the greater d50 size of the copper tails (85 microns compared with 5 microns for the coal tailings). The dashed exponential correlation provides an improved fit but still does not fully describe the concave shape.

The yield stress of the coal tailings is adequately correlated against Cv by either a Power Law or an Exponential equation (Figure 2). However for the copper tails, the correlation of the yield stress (Figure 3) is not so good. Figure 3 illustrates the less than ideal correlation of yield stress as a Power Law function of Cv for the copper tails. The situation is slightly improved if an exponential correlation is adopted but in both cases the correlation trends suggest poor extrapolation to higher concentrations. Similar poor correlations occur with many other slurries.

3. CONVENTIONAL CORRELATION OF PLASTIC VISCOSITY

Temperature typically has little influence on the yield stress but generally affects the plastic viscosity in a similar manner as for water. To facilitate comparisons with tests at different temperatures the plastic viscosity values are normalised to 20^{0}C by assuming plastic viscosity varies with temperature in the same manner as does the viscosity of water. At zero concentration the plastic viscosity should equal the viscosity of water which at 20^{0}C is 1 mPas.

As was discussed in Section 2, it is common for yield stress to be correlated against volume concentration, Cv. However it is generally found better to correlate plastic viscosity against Volume Ratio (Vr) of solids where Vr = ratio of volume of solids to volume of water = Cv/(1-Cv), see A.D. Thomas 2010. Figure 4 shows plastic viscosity versus Vr for the coal tailings of Figure 1. The exponential correlation shown provides a good fit (R^2 = 0.9949).

The authors have found that the plastic viscosity of most slurries is reasonably well correlated by an exponential function of Vr. For example, whereas the yield stress of the copper tails in Figure 3 was not correlated very well, the correlation of the plastic viscosity is much better as seen in Figure 5.

A.D. Thomas (2010) analysed test results for over 200 slurries and found that the exponential constant in the plastic viscosity correlation was a minimum 2.7 for very coarse slurries and increased as the d50 particle size decreased. Thus the 11.445 exponential constant value in for the coal tailings (d50 = 5 microns) in Figure 4 is higher than the 5.4328 for the coarser copper tails (d50 = 85 microns) in Figure 5. Both these values are consistent with that predicted by A.D. Thomas (2010) for the relevant d50 sizes.

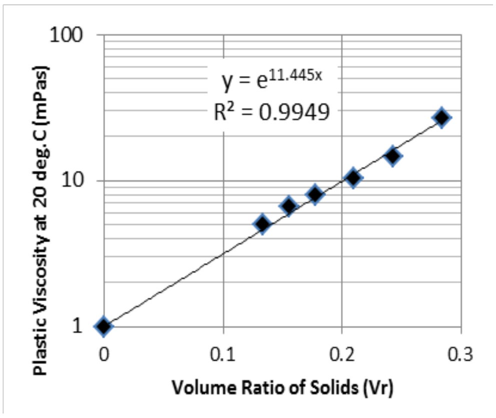


Fig. 4 Plastic viscosity coal tailings versus Vr

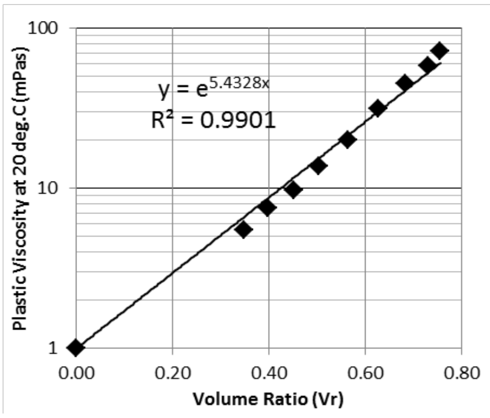


Fig. 5 Plastic viscosity copper tails versus Vr

4. PROPOSED NEW METHOD OF CORRELATING YIELD STRESS

Having considered conventional correlations methods we now introduce the proposed new method, firstly for yield stress. Consider the copper tails previously discussed in Figure 3. To produce this graph involved two columns of spread sheet data, Cv and Yield Stress and the fitting of Power Law and exponential trend lines. To apply the new correlating method, three columns are used, Cw, $Cw^{\mathbf{n}}$, and Yield Stress as seen in Table 1. Yield stress is graphed against $Cw^{\mathbf{n}}$ and the value of **n** is manually changed until the best fit is obtained. In this case **n** = 5.5 gave the best fit as seen in Figure 6.

For reporting, the yield stress data are now plotted against Cw with the correlation included as an additional series equal to 0.439407 exp(3.54526E-10 $Cw^{5.5}$). The resulting graph is shown in Figure 7. The correlation in Figure 7 can be compared with Figure 3 and is seen to be superior to the conventional correlations using Cv. Also, the dashed curves in Figure 7 are Exponential and Power Law trend lines applied directly to Cw (rather than to $Cw^{\mathbf{n}}$). The $Cw^{\mathbf{n}}$ correlation method once again gives a far better fit to the data.

	n=	5.5
Cw	Cw^n	Yield stress
67.19	1.122E+10	24.2
66.43	1.054E+10	17.8
64.98	9.339E+09	12.2
62.93	7.829E+09	6.84
60.51	6.310E+09	4.06
57.67	4.844E+09	2.6
55.03	3.744E+09	1.73
51.85	2.698E+09	1.05
48.5	1.869E+09	0.87

Table 1 Applying $Cw^{\mathbf{n}}$ method to yield stress of copper tails

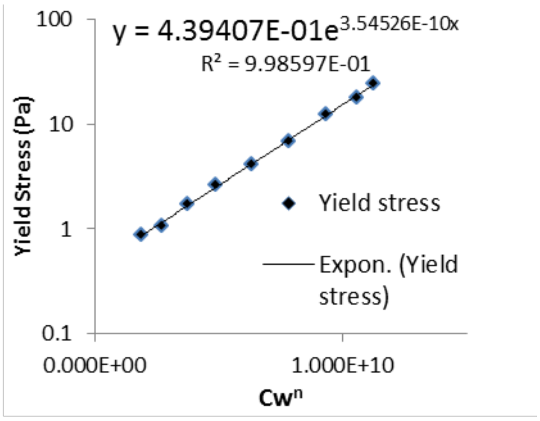
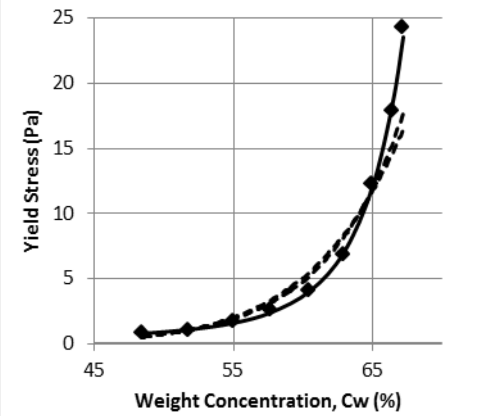


Fig. 6 Yield stress copper tails versus $Cw^{\mathbf{n}}$



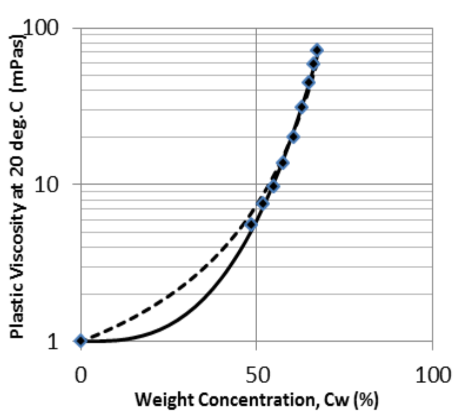


Fig.7 Yield stress copper tails versus Cw. Cw^n correlation compared with data

Fig. 8 Plastic viscosity copper tails versus Cw

5. NEW METHOD APPLIED TO PLASTIC VISCOSITY

A similar approach is followed with the plastic viscosity. In this case, for the copper tails, the best correlation is achieved with **n** = 2.9 with the exponential equation: Plastic viscosity = exp(2.10314 x 10^{-5} $Cw^{2.9}$). The R^2 value is 0.99766 which is higher than the R^2 in Figure 5 using the conventional Vr approach. The resulting graph is shown in Figure 8 and compared with the conventional Vr correlation, shown dashed, obtained from Figure 5 and transferred to plastic viscosity versus Cw.

6. RELATIONSHIP OF Cw^n METHOD TO CONVENTIONAL CORRELATION METHODS

The Cw^n method is quick and easy to use and almost always gives a better correlation of the data than the conventional methods. It is particularly useful for presenting test results in a report. However fundamentally the rheology of a slurry must depend primarily on the volume of solids rather than weight, and the Cw^n correlating method does not claim to be as theoretically rigorous as the conventional methods which are based on volume concentration, Cv, or Volume Ratio, Vr. Nevertheless the exponent **n** in the Cw^n method is related to the conventional correlations. For example the greater the degree of concavity in the Yield Stress versus Cv logarithmic plot (compare Figure 3 with Figure 2) the higher the value of n needed in the Cw^n method to achieve the best correlation. As discussed in Section 2.1, the coarser the slurry, the greater the concavity in the yield stress versus Cv plot. This means that **n** will increase as the slurry coarsens.

As an example of a very fine slurry, a minus 45 micron copper ore slurry (p80 = 33 microns, d50 = 16 microns, SG 3.11) gave a very good correlation between yield stress and Cv, with the Power Law exponent equal to 3.24 and R^2 = 0.9995 and with very little concavity in the yield stress versus Cv logarithmic plot. Applying the Cw^n method to this slurry, a selected value of n = 0.9 gave an even better correlation with R^2 = 0.9999. The coal tailings of Figure 2 with a Power Law exponent of 4.1544, R^2 = 0.9954, shows a slight concavity and required n = 1.2 for optimum correlation using the Cw^n method giving R^2 = 0.9999. Further increase in coarseness as for the coarser copper tails of Figure 3, with a Power Law exponent relating yield stress to Cv equal to 6.5974, the $Cw^{\mathbf{n}}$ method with **n** = 5.5, gives a much better correlation as seen in Figures 6 and 7. These three examples confirm that the degree of concavity of the yield stress versus Cv logarithmic plot increases as the slurry coarsens and the optimum value of **n** in the $Cw^{\mathbf{n}}$ method increases also. The optimum value of **n** also depends on the solids SG with n approximately doubling for example as the SG increases from 1.4 to 5.0.

7. CONCLUSIONS

A convenient method of correlating rheology data against weight concentration has been presented. The method is simple to use and consistently gives a better correlation than more conventional methods. The method is ideal for presenting rheology test results in reports. Although the method is not claimed to be as fundamentally correct as methods involving volume concentration or volume fraction of solids, it is shown that the parameters of the new method are related to the more fundamental parameter values.

8. REFERENCES

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