Hydrotransport 19 Conference, Sept 24-26, 2014, Colorado Springs, USA

Slurries of most interest to the mining industry flow homogeneously and the deposit velocity is the key parameter

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ABSTRACT

The great majority of slurries pumped in the mining industry flow homogeneously without exhibiting any heterogeneous "hook" at low velocities. Particle size distributions of 150 concentrates and tailings representative of these types of slurries are presented. The key parameter is the deposit velocity and available methods of prediction are compared, especially in regard to the variation of deposit velocity with pipe diameter. Variation of deposit velocity with concentration is discussed, as is the role of the laminar-turbulent transition at higher concentrations. A comparison example in an operating 593 mm ID pipeline indicates that the inherent viscosity, as introduced by the author previously, is the appropriate viscosity to use.

1. INTRODUCTION

Slurry pipeline hydraulics has been studied seriously since the 1950's. The majority of research has been involved with heterogeneous, settling type slurries, typically studied using near mono-sized sands in water. For these slurries, on a log-log plot of pressure gradient versus velocity, the pressure gradient progressively deviates away from the straight line water curve as the velocity decreases, with deposition eventually observed along this heterogeneous curve. In the current paper the deposit velocity, Vd, is defined as the velocity at which a stationary bed of solids first appears. Durand (1) was among the first to present a method to predict the heterogeneous pressure gradient and the deposit velocity. In 1972 Wilson introduced his sliding-bed theory which was refined with co-workers up until the present day with the latest prediction methods described in the book by Wilson, Addie, Selgren and Clift (2). Pipeline hydraulics for these settling slurries depends on particle size, solids and fluid density, fluid viscosity and solids concentration.

Non-settling slurries such as clay slurries have also been studied as non-Newtonian fluids, usually using the Bingham plastic model. Pipeline prediction is based on slurry rheology determined in laboratory-scale viscometer tests or small-scale pipe loop tests.

However the majority of slurries pumped in the mining industry are not represented by these two distinct types of slurries. During mineral processing the ore is subjected to comminution, with the resulting concentrate and tailings slurries possessing a wide particle size distribution. Also it turns out that in most cases, to release the valuable minerals from the ore, the top size generally needs to be ground to less than about 300 μm. This means that, with the wide size distribution, the finer particles are approaching colloidal size. These slurries possess non-Newtonian properties and flow as pseudo-homogeneous fluids but with some settling tendency. On a logarithmic plot, the slurry pressure gradient tends to parallel the water line right down to the deposit velocity. This means that once the deposit velocity is determined, an operating velocity can be selected at a suitable margin above the deposit velocity. The prediction of the pressure gradient is then relatively straight forward, based on pseudo-homogeneous behaviour. Exceptions to the above generalisations include slurries in the oil sands industry, dredging industry, mineral sands, and coarse rejects in the coal industry, which require analysing using the Durand method or more properly, Wilson type analyses. These heterogeneous slurries are not discussed here further.

Particle size distributions of concentrates and tailings, test loop data from Schriek *et al* (3) and in-house data of the author are presented in this paper to illustrate homogeneous behaviour. The most relevant methods of predicting the deposit velocity are those of D.G. Thomas (4), Wasp *et al* (5), Wilson and Judge (6), A.D. Thomas (7), Sanders *et al* (8) and Thomas and Fitton (9). Predictions for these are compared in detail for pipe sizes ranging from 10 mm to 1000 mm and mean particle sizes from 500 μm to 10 μm. It is recommended that pressure gradients at the determined operating velocity be determined by the method of Wilson and Thomas (10).

2. PARTICLE SIZE DISTRIBUTIONS AND HYDRAULIC BEHAVIOUR OF TYPICAL MINERAL SLURRIES

Figure 1 shows particle size distributions of 39 concentrates with which the author has been involved over the past 30 years, either with slurry pipeline design or review, or through laboratory testing. The concentrates include copper, zinc and lead concentrates and magnetite iron ore slurries. All are suitable for long distance pipeline transport. Some are already pumped in existing long distance pipelines and all are known to flow in a homogeneous manner at velocities down to the deposit velocity. Figure 1 also includes the size distribution of an iron ore slurry tested by Saskatchewan Research Council (SRC) (3).

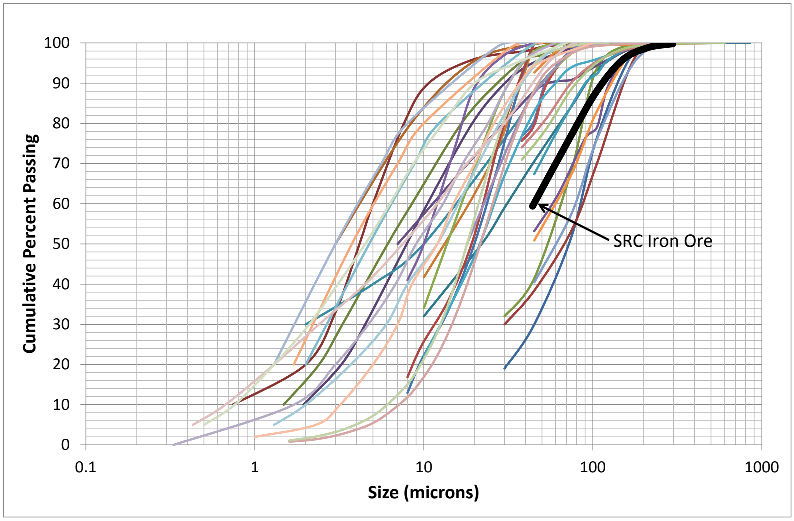

Figure 1 Particle size distributions of 39 concentrates

Figure 2 shows particle size distributions of 105 different tailings slurries with which the author has been involved over the past 30 years, either with pipeline design or review, or simply through laboratory testing. The tailings include copper, zinc, lead, gold, nickel, uranium, and diamond tails. The tailings are coarser than the concentrates of Figure 1 but the solids also have lower relative densities (specific gravity, SG) so the majority are also expected to flow homogeneously.

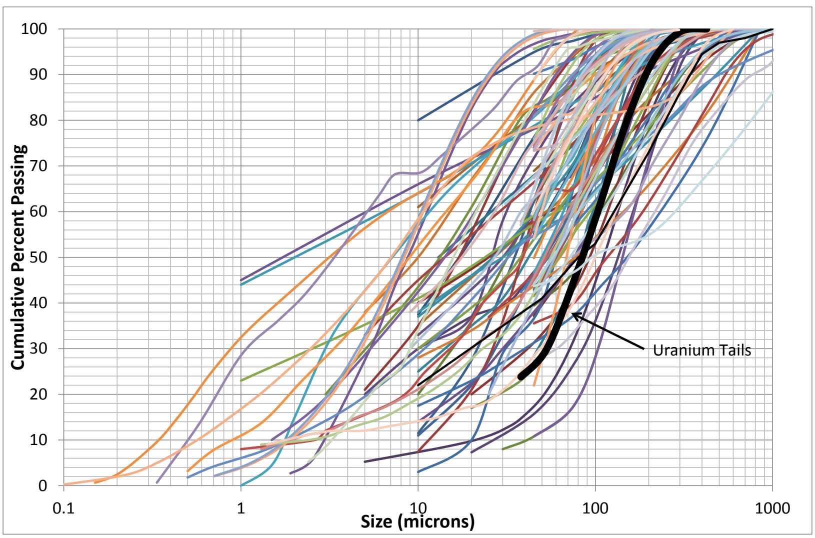

Figure 2 Particle size distributions of 105 tailings slurries

Figure 3 shows pressure gradient versus velocity data for the SRC iron ore (3) of Figure 1 at two concentrations in a 315 mm ID pipe. The lower line is for water. The 57% concentration (by weight) data parallel the water curve on the logarithmic plot. The vertical line indicates Vd at 1.58 m/s. Deposition occurs under pseudo-homogeneous flow conditions, with no evidence of any heterogeneous "hook" in the pressure gradient at the lower velocities. At the higher 69% concentration, the high velocity data parallel the water curve but at lower velocities the slurry pressure gradient decreases slightly towards the water curve. This suggests non-Newtonian behaviour, approaching a transition velocity probably around 0.8 m/s. The reduction in pressure gradient approaching transition is predicted by the theory of Wilson and Thomas (10). No deposition was reported by Schriek *et al* (3) at the lowest velocity (0.93 m/s) for the 69% concentration. The fact that the SRC iron ore is amongst the coarsest of the concentrates and also has about the highest solids SG (5.245) suggests that all the concentrates in Figure 1 will flow homogeneously.

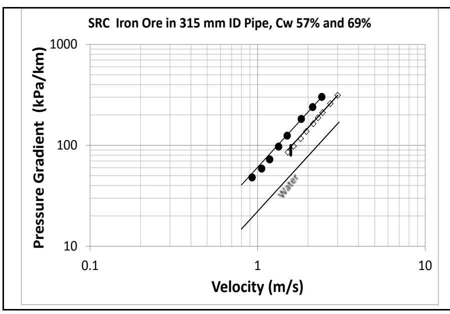

Figure 3 Pressure gradient versus velocity, SRC iron ore

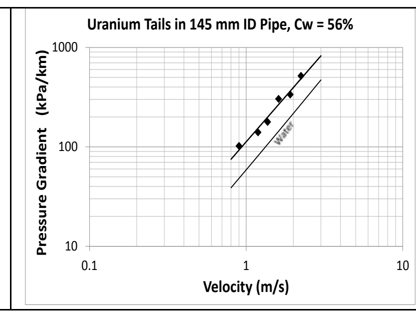

Figure 4 Pressure gradient versus velocity, uranium tails

Figure 2 includes the size distribution for Uranium Tails, SG 3.3, (in-house data). Figure 4 shows pressure gradient versus velocity data for the uranium tails in a 145 mm ID pipe at 56% concentration. A stationary bed was present at the lowest velocity (0.90 m/s). The slurry pressure gradient essentially parallels the water curve, indicating pseudo-homogeneous behaviour. The size distribution of the uranium tails is coarser than the majority of the tails and has a higher solids SG (3.3) than the typical SG 2.7 for tails. Taking this into account it can be concluded that pseudo-homogeneous behaviour would apply to the majority of the tailings slurries in Figure 2, probably all those with a p80 size less than about 300 μm (d50 size less than about 150 μm). This suggests that for the majority of tailings, the pressure gradient can be predicted using a non-Newtonian homogeneous slurry approach.

3. DEPOSITION CRITERIA FOR PSEUDO-HOMOGENEOUS FLOW

3.1 Early Work from 1950's to 1970's

Durand (1) was among the first to present a method to predict the deposit velocity. He was mainly concerned with relatively narrow-size coarse sands and gravels in water flowing in a heterogeneous or sliding bed mode. His work is not directly relevant to the pseudo-homogeneous flow of interest in the current paper but is included to illustrate trends. His well-known classic equation is given below:

$$Vd = F_L [2 g D (S-1)]^{0.5} \qquad (1)$$

F_L was given graphically as a function of particle size and concentration. Eqn 1 indicates that Vd varies as the square root of pipe diameter, regardless of particle size. D.G. Thomas (11) studied similar large-particle slurries and found that in terms of friction velocity, Vd^* varied with $D^{0.6}$, once again regardless of particle size. D.G. Thomas (4) also investigated the deposit velocity criteria for slurries with particles finer than the viscous sub-layer and presented the following equation in terms of friction velocity at a low concentration, V_{d0}^*. He gave a separate equation for the effect of concentration. Note that pipe diameter is absent from Eqn 2 except indirectly through the friction velocity. Hence Eqn 2 indicates that V_d varies with pipe diameter approximately to the power 0.1.

$$W/V_{d0}^* = 0.0083 (d V_{d0}^* \rho / \mu)^{2.61} \qquad (2)$$

Wasp *et al* (5) identified the role of laminar-turbulent transition in determining the deposit velocity for fine particle, non-Newtonian, slurries. For these slurries they equated Vd to the transition velocity. They illustrated how, using equation (3) below of D.G. Thomas (12) for transition velocity for a Bingham plastic, the transition velocity depended only on the Bingham yield stress and slurry density, and hence the transition-based deposit velocity is independent of pipe diameter.

$$V_T = 19 (\tau_y /\rho)^{0.5} \qquad (3)$$

For coarser slurries Wasp *et al* presented a variation of the Durand equation (Eqn 1) from Wicks (13). Their Eqn 4 below includes a term involving particle size and also indicates that Vd varies with $D^{0.33}$ rather than $D^{0.5}$. F_L' is a function of concentration.

$$Vd = F_L' [2 g D (S-1)]^{0.5} (d/D)^{1/6} \qquad (4)$$

Figure 5 summarises the above early work in regards to the influence of pipe diameter on the deposit velocity for sand in water. The top three lines are Durand predictions using Eqn 1 for 0.5mm, 0.2 mm and 0.1 mm sand. According to Durand, as the particle size decreases the deposit velocity decreases but the slope of the lines remains constant and equal to 0.50. The lower full line is the predicted deposit velocity based on Eqn 2 of D.G. Thomas (4). Note that Eqn 2 was developed for flocculated particles, and it will be shown later how the predicted curve in Figure 6 is actually too high for sand in water. But it is included to conveniently illustrate the slope variations.

Wasp *et al* (5) were involved in wide size distribution slurries with particle sizes ranging down to micron sized so it is not surprising that they found an intermediate value of n = 0.33 provided the best fit to their data as indicated by the three dashed line predictions for the same three particle sizes used in Eqn 4. In this case as the particle size decreases the deposit velocity decreases but the slope of the lines remains constant and equal to 0.33. So the top three full-lines and the lower full-line illustrate how the slope n changes from 0.5 to approximately 0.1 as the particle size decreases. However these early correlations provide no information as to how the slope might gradually change with particle size.

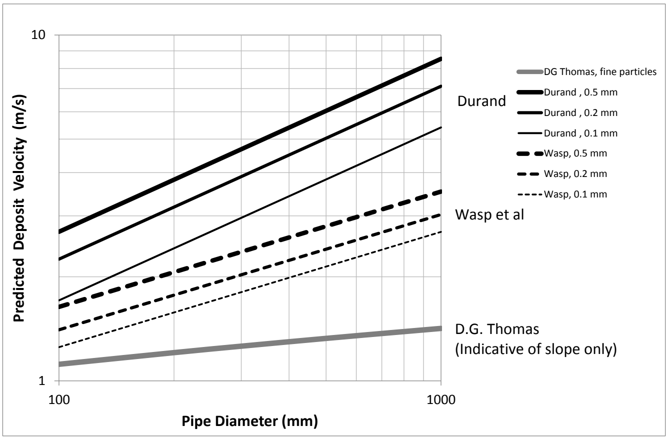

Figure 5 Comparing predicted slope variations with pipe diameter, sand in water, Durand, Wasp et al, D.G. Thomas

The above trends apply to particles in Newtonian fluids. With typical wide-size distribution slurries there are sufficient micron size particles to give the slurry non-Newtonian properties. For these slurries at higher concentrations the deposit velocity may be equated to the laminar-turbulent transition velocity as pointed out by Wasp *et al* (5). In this case using Eqn 3 the deposit velocity is independent of pipe diameter and would be a horizontal line in Figure 6. The role of transition in determining deposit velocity is discussed later in Section 4.4.

3.2 Work in the Late 1970's

Figure 5 illustrates that, to fully represent the situation, a prediction method should allow not only for a reduction in Vd as particle size decreases, but also should allow for a continuous reduction in the slope of lines in the Vd versus D plot. In 1976 Wilson and Judge (6) provided just such a prediction method. Their theory allowed for a reduction in Vd as turbulence supported a greater proportion of particles resulting in a modified F_L in the Durand equation (Eqn 1) given by:

$$F_L = 2.0 + 0.3 \log_{10}\Delta \qquad (5)$$

where $$\Delta = 0.75\, W^2/[g\, D\, (S-1)] = d/(DC_d) \qquad (6)$$

W is the particle settling velocity of size d in a quiescent fluid (calculated here as for a sphere) and C_d is the drag coefficient of the particle. Eqn 5 was valid for $10^{-5}<\Delta< 10^{-3}$. Wilson and Judge (14) later provided a nomograph for Vd based on the above. Wilson later provided an additional nomograph expanding the prediction to materials with SG other than 2.65 - see Wilson et al (2).

A.D. Thomas (7) conducted loop tests on a range of slurries including very fine particle size slurries, and observed that as the particle size became finer, Vd did not continue to decrease but reached a lower limit. He developed a prediction method for Vd for particles smaller than the viscous sub-layer, based on the Wilson sliding bed theory.

$$V^*_d = 1.1\,[g\,\mu\,(\rho_p - \rho)/\rho^2]^{1/3} \qquad (7)$$

Pipe diameter is not included in Eqn 7 other than indirectly through the friction velocity. For large pipe diameters, Eqn 7 indicates Vd is approximately proportional to $D^{0.1}$. Note also that particle size does not appear in Eqn 7. As long as the particle size is less than the viscous sub-layer thickness, V^*_d is predicted to be the same for all particle sizes. Eqn 7 in effect provides a lower limit for Vd as particle size is reduced. The previous Eqn 2 of D.G. Thomas (4) includes a parameter d which is the floc size. A.D. Thomas (7) showed that Eqn 2 is consistent with Eqn 7 if particle size rather than floc size is used.

The full lines in Figure 6 show the predicted Vd versus D for sand in water, based on the Wilson and Judge (6) Eqn 5 for particle sizes 500, 300, 200, 150, 100, 75 and 60 μm together with the maximum and minimum Vd curves. Note how the slope of these curves decreases as the particle size decreases, thereby providing the required transition in slope discussed in relation to Fig. 5. Note also how, for any given particle size, the slope decreases as the pipe size increases. So the predictions of Wilson and Judge are shown to provide the required transition, but the minimum value of $\Delta = 10^{-5}$ imposed on Eqn 5 results in the indeterminate region shown in Figure 6. Between the limits of the Eqn 5 predictions and the minimum set by Eqn 7, no prediction is available.

From the nomograph of Wilson and Judge (14), a maximum value of Vd, (termed by Wilson and Judge the *maximum maximorum*), can be determined which provides an upper limit to Vd. The slope of Wilson and Judge's *maximum maximorum* curve is 0.6, the same as the slope given by D.G. Thomas (11). A lower limit is set by Thomas's (7) Eqn 7. The slope of the minimum curve (Eqn 7) varies from approximately 0.15 at D = 10 mm to approximately 0.10 at D = 1000 mm.

In Figure 6 the predictions using the nomograph presented by Wilson and Judge (14) for particle sizes 500, 300, 200 and 150 μm are shown as dashed curves. The nomograph is limited to a minimum 100 mm pipe and minimum 150 μm particle size, with the 150 μm particle size limit due to the same 10^{-5} minimum on Δ as noted in connection with Eqn 5. Therefore the nomograph does not provide any solution to predicting Vd in the indeterminate region.

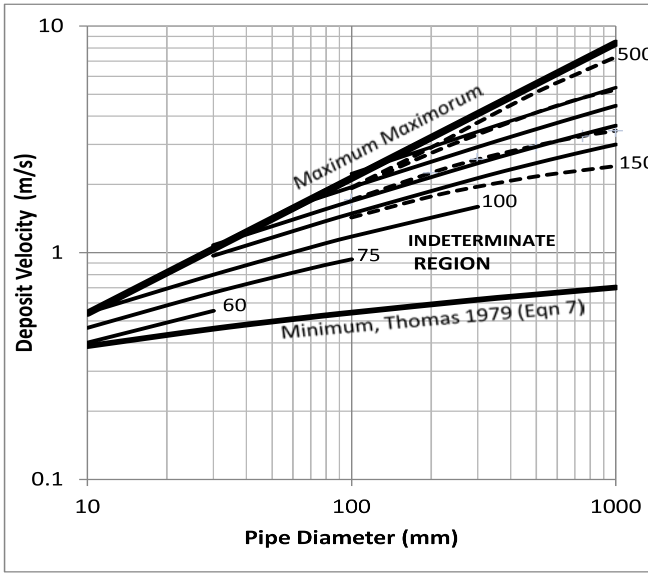

Figure 6 Deposit velocity predictions for sand in water,
full thin lines - Wilson and Judge (6), dashed lines - Wilson and Judge (14)

A.D. Thomas (7) recognised the indeterminate region created imposed by the Wilson and Judge (6) restriction of Eqn 5 to $\Delta < 10^{-5}$ and proposed that Vd be determined by a combination of Vd^*_Δ given by Eqn 5 and Vd^*_δ given by Eqn 7 with Δ_{min} 2.15 x 10^{-7}.

$$Vd^* = [(Vd^*_\Delta)^2 + (Vd^*_\delta)^2]^{0.5} \qquad (8)$$

Predictions using Eqn 8 gives realistic trends for d50 = 100 μm, 75 μm and 60 μm out to pipe diameters of 1000 mm, 500 mm and 200 mm respectively. However the trends for predictions for particles finer than 50 μm are unrealistic.

3.3 Most Recent Work

Further investigation into the "Indeterminate Region" between the predictions of Wilson and Judge (6) and the lower limit of Thomas (7) had to wait 20 years until the work of Gillies *et al* (15) and Sanders *et al* (8) was reported. The latter workers modified the Thomas (7) approach to arrive at the following expression for deposit velocity:

$$Vd^* = (0.76 + 0.15\, d^+)/ [(C_{max} - C_r)^{0.88}] \qquad (9)$$

where d^+ $= d\, \rho_f V_d\, (f/2)^{0.5}/ \mu_f$
C_{max} = maximum settled volume of particles, fraction
C_r = mean in-situ solids concentration, volume fraction

The dashed curves in Figure 7 shows predictions using Sanders *et al* (Eqn 9) for sand in water for an assumed C_{max} = 0.6 and C_r = 0.2. The predicted curves are for d = 150, 100, 75, 50, 40, 30, 20 and 10 μm. For comparison the four thick grey curves near the top of the graph are from the Wilson and Judge (14) nomograph predictions for d = 500, 300, 200 and 150 μm. The thin full-line curves are Wilson and Judge (6) predictions for d = 500, 300, 200, 150, 100, 75 and 60 μm.

Although the predictions of Sanders *et al* (8) do cover the "Indeterminate Region" of Figure 6 there are two limitations with these predictions. Firstly since pipe diameter is not included in Eqn 9 other than indirectly through the friction velocity, the variation of V_d predicted using Eqn 9 is similar to that of Eqn 7. Hence the Sanders *et al* predictions (Eqn 9) parallel the Minimum Vd, (Thomas, 7), Eqn 7 prediction curve and so do not allow for the required increase in slope as the particle size increases which is expected from the previous arguments relating to Figure 6, and which is predicted by Wilson and Judge (6), Eqn 5, and is also predicted using the Wilson and Judge nomograph (14).

A second limitation of Eqn 9 is that it is really only applicable to bimodal particle size distribution slurries and not to slurries with a continuous size distribution. Eqn 9 predicts an increase in Vd with concentration increase whereas, as noted by the authors themselves, it is known that for many wide-size distribution slurries, which possess significant rheology, V_d can decrease with concentration increase. Nevertheless regardless of these two limitations, the method of Sanders *et al* may be useful for particle sizes below about 50 μm where the predicted Vd is only slightly higher than that given by Thomas (7). For these fine particle sizes the lack of slope steepening is not a serious problem.

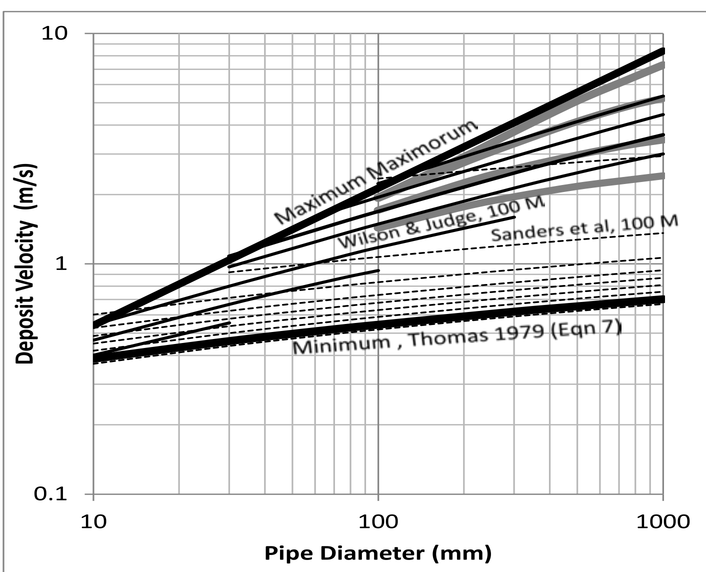

Figure 7 Deposit velocity predictions for sand in water, full thin lines - Wilson and Judge (6), full thick lines - Wilson and Judge (14), dashed lines - Sanders et al (8).

Thomas and Fitton (9) presented a method (Eqn 10) which gives straight lines of varying slope approximating the likely extrapolations of the Wilson and Judge (6) Figure 6 predictions into the "Indeterminate Region". The relevant particle settling velocities (based on a sphere) used in the Wilson and Judge predictions for the three finest particle sizes shown in Figure 6, are 60 μm (W = 3.23E-3 m/s), 75 μm (W = 5.01E-3 m/s) and 100 μm (W = 8.41E-3 m/s). Thomas and Fitton assumed the predicted curves apply to other particle size solids of different solids SGs and viscosities, provided the same values of W apply.

$$V_d = A\ D^n \tag{10}$$

where $A = 25.71\ [\ (\rho_s - \rho_w) / \rho_w / 1.65\ ]^{0.5}\ W^{0.5}$ (11)

$n = 1.42\ W^{0.35}$ (12)

W is the settling velocity of a single particle assumed to be a sphere. These equations are not non-dimensional and SI units apply to all parameters. The tailings of interest to Thomas and Fitton had particle size distributions similar to those of Figure 2 and the weighted-mean particle size was assumed to be the relevant particle size. If Vd predicted by Eqn 10 is less than Vd predicted by Eqn 7 then the prediction of Eqn 7 applies.

Figure 8 compares the predictions of Thomas and Fitton (Eqn 10) for sand in water with Wilson and Judge (6). The Thomas and Fitton predictions are shown dashed and relate to the particle sizes (d) in microns shown in the RHS of the Figure. The thin full curves are Wilson and Judge (1976) predictions for d = 500, 300, 200, 150, 100, 75 and 60 μm.

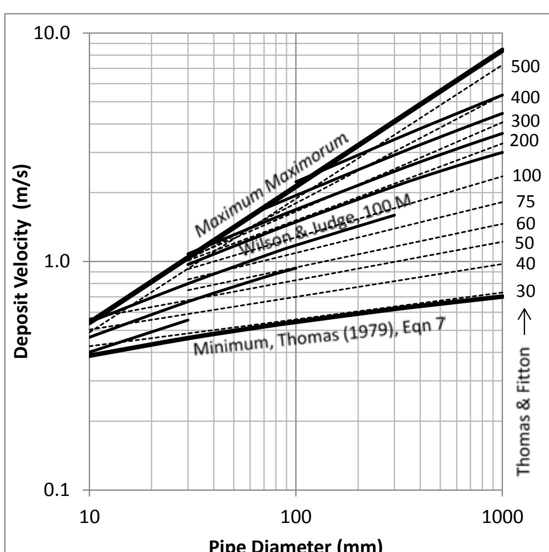

Figure 8 Deposit velocity predictions for sand in water, dashed lines - Thomas and Fitton (9), full lines – Wilson and Judge (6)

The predictions of Thomas and Fitton (9) shown in Figure 8 are straight lines on the log-log V_d versus D plot, with the slope decreasing as the particle size decreases so the method does allow for the reduction in slope from the Durand 0.5 slope to the A.D. Thomas (7) 0.1 slope as the particle size decreases. However the method does not allow for the reduction in slope as the pipe size increases, which is a feature of the Wilson and Judge predictions. The Thomas and Fitton predictions in Figure 8 apply to sand in water but the equations involve the settling velocity W, and so can be applied to other solids SGs and viscosities.

4. TRANSLATING SAND-IN-WATER PREDICTION METHODS TO WIDE SIZE DISTRIBUTION SLURRIES

4.1 Using Inherent Viscosity to Calculate Particle Settling Velocities

All of the previous discussion and relates to narrow size distribution sand in water. Typical industrial slurries have wide size distributions similar to those in Figures 1 and 2. Use of the weighted-mean particle size is one method of accounting for the wide size distribution. These slurries also generally possess non-Newtonian characteristics and are described in this paper by the Bingham plastic model. The Bingham yield stress and plastic viscosity have been determined for all of the concentrates and tailings in Figures 1 and 2 in rheology tests conducted on the total slurry using a rotational viscometer. This is a different situation than for a bi-modal slurry such as a clay slurry with sand particles added. In that case the rheology of the clay slurry is measured and the plastic viscosity of the clay “vehicle” slurry is the appropriate viscosity to use in calculating particle settling velocities of the sand portion. With wide size distribution slurries the problem is to determine what is the “vehicle” slurry portion. Some authors have selected minus 75 μm or minus 45 μm as the criterion for the vehicle slurry but this approach would seem rather arbitrary.

A.D. Thomas (16) introduced the concept of an inherent viscosity and argued that the relevant viscosity to calculate the particle settling velocity and predict the deposit velocity is the inherent viscosity. He assumed the plastic viscosity of the total slurry, as measured in the viscometer, is a function of Volume Ratio (Vr) of the solids where Vr = Cv/(1-Cv), Cv being the volume concentration of solids expressed as a fraction.

$$\mu_{tot} = e^{AtotVr} \tag{13}$$

He then argued that the total viscosity is composed of an inherent viscosity portion multiplied by a mechanical interference component which could be described by $\mu_{mech} = e^{2.7\ Vr}$. Hence Atot is determined from the measured plastic viscosity of the total slurry via Eqn 10 and the inherent viscosity is then given by:

$$\mu_{inh} = e^{(Atot - 2.7)Vr} \tag{14}$$

The rationale for use of the inherent viscosity rather than the total viscosity is that all the prediction methods for deposit velocity, including the method of Wilson and Judge, use the viscosity of water. They do not use the total viscosity of the water-sand mixture. In effect the viscosity of water is the inherent viscosity of the sand-water slurry.

4.2 Data from Operating a 593 ID Pipeline Justifies use of Inherent Viscosity

A.D. Thomas *et al* (17) described the tailings pumping system of the Boddington gold mine in Western Australia. Dual 593 mm ID pipelines transfer tailings 4.8 kms from the processing plant to a booster station. The lowest operating flow rate per pipeline is 1407 m^3/h at 65% solids concentration which represents a velocity of 1.41 m/s. Slurry properties are: mean particle size 65 μm, solids SG 2.83 and plastic viscosity at 65% concentration 25 mPas.

Following the procedure outlined in Section 4.1, the calculated value of A_{tot} = 4.91 so A_{inh} = 2.21 and μ_{inh} = 4.26 mPas compared with the measured μ_{tot} = 25 mPas. Calculated particle settling velocity, W, is 9.9E-4 m/s based on μ_{inh} = 4.26 mPas, giving Δ = 6.9E-8 which is much less than the 1E-3 limit set by Wilson and Judge (6). Hence V_d is in the “indeterminate region” of Figure 6. Based on W, the equivalent sand in water particle size is 33 μm which indicates Vd close to the Thomas (7) minimum in Figures 6, 7 and 8. Table 1 compares Vd predictions using Thomas (7), Sanders *et al* (8) and Thomas and Fitton (9). In regard to the Thomas and Fitton predictions, both are less than the Thomas (7) prediction so the latter applies as indicated by the number in brackets.

Table 1
Comparing Predictions using Inherent Viscosity and Total Measured Viscosity

	Using μ_{inh}	Using μ_{tot} (measured viscosity)
Mean Particle Size (□m)	65	65
Viscosity (mPas)	4.26	25
Particle Settling Velocity (W) (m/s)	9.9E-4	1.7E-4
<u>Predicted Vd</u>		
A.D. Thomas (7)	1.02	1.59
Sanders et al (8)	1.20	2.04
Thomas and Fitton (9)	0.80 (1.02)	0.34 (1.59)

When using the total viscosity, all three prediction methods predict Vd higher than the minimum operating velocity of 1.41 m/s even though no operational problems are experienced when operating at the minimum 1.41 m/s. If inherent viscosity is used, all three methods predict Vd comfortably less than 1.41 m/s, consistent with the operational experience. Although this comparison is not definitive it does support the use of inherent viscosity. Indeed part of the impetus for Thomas’ (16) paper was the observation over many years that as the concentration increases and the total viscosity rises exponentially, the A.D. Thomas (7) prediction gave increasingly high and unrealistic values for Vd. The Thomas (7) theory was of course originally developed based on experimental data for solids in fluids of known viscosity rather than on the viscosity of the total slurry.

4.3 Weighted-Mean Particle Size and Plastic Viscosity of Concentrates and Tailings

The average weighted-mean particle sizes of the concentrates and tailings of Figures 1 and 2 have been determined together with an average value for the viscosity parameter A_{tot}. The coarsest mean size and its associated A_{tot} has also been determined. Assuming a typical pumping concentration of 60% the inherent viscosity and hence the settling velocity, W, of the mean particle size is calculated. Back calculation from this W enables an equivalent particle size for SG 2.65 sand in water to be determined and compared with the particle sizes in Figure 6. Table 2 summarises these calculations.

Table 2
Calculating d_{equiv} for sand in water, for slurries of Figures 1 and 2

	dm (μm)	SG	μ_{inh} at 60% (mPas)	W (m/s)	d_{equiv} for sand in water (μm)
Concentrates					
Average	27.3	4.70	3.06	4.9E-4	23
Coarsest	77	4.74	1.83	6.59E-3	113
Tailings					
Average	69.2	2.95	11.9	4.3E-4	22
Coarsest	272	2.60	5.88	1.07E-2	128

The d_{equiv} values range from 22 μm to 128 μm. It is clear from Figure 6 that this particle size range lies in the "Indeterminate Region" for typical pipe diameters. Hence in the exact region of most interest for the majority of industrial slurries there is no clear method of predicting Vd. The methods of Sanders et al (8) and Thomas and Fitton (9) do cover this region but each method has limitations.

4.4 Laminar-Turbulent Transition

For the wide particle size distribution slurries of interest here, both the plastic viscosity and the yield stress increase as the concentration increases. But the yield stress generally increases at a faster rate than the plastic viscosity. It has long been known that the yield stress largely determines the laminar-turbulent transition velocity as indicated by Eqn 3 of D.G. Thomas (12). More recently Wilson and Thomas (18) derived a similar equation but with a value for the constant of 25 for large pipe sizes, rather than 19 (see Eqn 3).

Since yield stress increases faster than plastic viscosity as the concentration increases, eventually the transition velocity, Vt, becomes greater than Vd determined by the other methods discussed in this paper. It is generally agreed that laminar flow without deposition can only be sustained if the pressure gradient is greater than about 2000 Pa/m. This is far higher than the pressure gradients applying to the slurries of most industrial interest. For example the pressure gradient of the Boddington slurry discussed in Section 4.2 is only 52 Pa/m. Hence, for almost all slurries of interest, once Vt increases above Vd then deposition will coincide with Vt.

4.5 Predicting the Pressure Gradient

Once the deposit velocity is predicted, a velocity margin, often around 0.3 m/s, is added to give the operating velocity. Almost all the slurries discussed in this paper flow pseudo-homogeneously in that the pressure gradient approximately parallels the water pressure gradient on a log-log plot. A non-Newtonian pressure gradient prediction method such as that of Wilson and Thomas (10) is appropriate.

5. CONCLUSIONS

The historical development of methods to predict the deposit velocity of slurries has been outlined and discussed in relation to 150 concentrate and tailings slurries with size distributions shown in Figures 1 and 3. The comparison among prediction methods focusses particularly on the variation of Vd with pipe diameter. The power law exponent of D is shown to decrease from about 0.5 for coarse slurries to about 0.1 for fine slurries. The method of Wilson and Judge (6) provides the most realistic variation of Vd with both D and particle size, but unfortunately does not extend to the fine particles of most interest to the industrially important slurries. The methods of Sanders *et al* (8) and Thomas and Fitton (9) do cover this fine particle region but both methods have limitations. Comparisons with operating data from a 593 mm ID pipeline support the author's contention that the appropriate viscosity to use in the prediction methods is the inherent viscosity as introduced by Thomas (16).

6. NOMENCLATURE

A	Given by Eqn 11
A_{tot}	Exponential constant in Eqn 13 for total viscosity
A_{inh}	Exponential constant in Eqn 14 for inherent viscosity = A_{tot}-2.7
C_d	Particle drag coefficient
C_{max}	Maximum settled volume concentration – see Eqn 9
C_r	In-situ volume concentration – see Eqn 9
Cv	Volume concentration of solids (fraction)
D	Internal pipe diameter (m)
d	Particle size (m)
d^+	Dimensionless particle diameter – see Eqn 9
dm	Weighted mean particle size
F_L	Durand parameter in Eqn 1
g	Gravitational constant (m/s^2)
n	Exponent of D on Vd versus D plot
SG	Solids specific gravity ($\rho_{solids}/\rho_{water}$)
Vd	Deposit velocity (m/s)
Vd^*	Friction velocity at Vd (m/s)
Vd^*_Δ	Friction velocity at Vd calculated using Wilson and Judge (1976), (m/s)
Vd^*_δ	Friction velocity at Vd calculated using Thomas (1979), (m/s)
V_{d0}^*	Friction velocity at deposition at infinitely low concentration, Eqn 2, (m/s)
Vr	Volume fraction of solids = Cv/(1-Cv)
W	Particle terminal settling velocity in quiescent fluid (m/s)
ρ	Density of fluid (kg/m^3)
ρ_p	Density of particle (kg/m^3)
μ	Viscosity of fluid (mPas)
μ_{tot}	Total (measured) viscosity of a slurry (mPas)
μ_{inh}	Inherent viscosity of a slurry (mPas)
τ_y	Bingham yield stress (Pa)

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