

A REVIEW OF DEPOSIT VELOCITY PREDICTION METHODS FOR MEDIUM PARTICLE SIZE SLURRIES IN PIPES

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SUMMARY

Methods of predicting the deposit velocity for wide particle size slurries with maximum particle size up to about 1 mm and maximum d_{50} size around 0.3 mm are outlined. These slurries generally possess non-Newtonian properties, typically modelled as Bingham plastics, and typically flow pseudo-homogeneously in turbulent flow down to the deposit velocity. Because they flow pseudo-homogeneously, pressure gradient prediction is relatively easy once a suitable operating velocity is selected. Consequently, the deposit velocity is the most important parameter as it determines the operating velocity.

Beginning with Durand and Condolios [6], the historical development of the major methods for predicting the deposit velocity for mono-size particles in water are first reviewed, and their advantages and limitations discussed. Methods of extending predictions to mono-size particles in viscous Newtonian fluids are then reviewed.

Next, prediction techniques relevant to deposition are reviewed for non-Newtonian slurries. These include prediction of the transition velocity between laminar and turbulent flow, and the critical pressure gradient required to prevent deposition under laminar flow conditions.

Finally, these prediction techniques are combined to apply to minus 1 mm, wide size distribution, viscous slurries, commonly encountered in the mining industry. Two deposit velocity prediction techniques for these types of slurries are discussed. The first technique, based on determining the inherent viscosity of the slurry and assuming the weighted mean particle size of the total slurry represented the relevant coarse particle dimension, was found to predict performance in an operating 593 mm ID pipeline. The second technique, based on assuming the minus 75 μm portion represented the “carrier” fluid and assuming the median size of the plus 75 μm portion represented the relevant coarse particle dimension, was found to give very good predictions of the observed deposition trends of Goosen and Paterson [8] for a minus 300 μm gold tailings tested in 100 mm, 152 mm and 242 mm test loops.

KEY WORDS: Slurry pipeline, Deposition, Pseudo-homogeneous, Laminar, Turbulent, Transition

1. INTRODUCTION

The pipe flow of slurries is important in many industries, especially in the mining industry, but also in the dredging, power generation, agricultural, sewerage and other industries. A slurry is here defined as a mixture of non-dissolvable solids in a liquid, with the liquid usually being water. The particle size can vary widely, from clays composed of colloidal-size particles less than 1 μm to coarse lumps of over 100 mm. However, the current paper focuses

on slurries with particle sizes ranging from colloidal-size to a maximum of no more than 1 mm, with a maximum median (d_{50}) size of no more than 0.3 mm. Such slurries are very common in many industries, especially in mining industry processes involving comminution [27].

An important parameter in slurry pipeline flow is the deposit velocity, V_d , which is defined here as the velocity at which a stationary bed of solids first appears at the bottom of the pipe as the velocity is progressively reduced. A wide size distribution slurry with a maximum particle size of no more than 1 mm, will generally flow pseudo-homogeneously in turbulent pipe flow and the deposit velocity is the most important parameter [27]. It is desirable to be able to predict V_d based on the fundamental properties of the slurry such as particle size distribution, solids and fluid density, solids concentration and slurry viscosity. If the proportion of colloidal-sized particles is significant, the slurry may have non-Newtonian rheological properties with the flow properties being described by a Bingham plastic model for example. In this case, this paper may refer to the “rheology” rather than “viscosity” of the slurry.

The pipe flow of slurries has been studied extensively over the last 100 years. The majority of the work can be separated into two groups. Group 1: studies of settling slurries composed of narrow size distribution, discrete particles in water. Group 2: studies of non-Newtonian colloidal or part colloidal type slurries in laminar or turbulent pipe flow. The most relevant works in these two groups relating to deposit velocity prediction are reviewed in this paper. More recently wide particle size distribution slurries have been studied and slurries composed of particles ranging from colloidal to mm size, are discussed in Section 4.

2. NARROW SIZE DISTRIBUTION DISCRETE PARTICLES IN WATER

2.1 Early Correlations

Group 1 studies involve the study of discrete particles in water, with all the early models assuming mono-sized particles with the size approximated by the median (d_{50}) size, for sands of narrow particle size distribution. Following [44], a narrow size distribution or grading, is defined here as one with $d_{85}/d_{50} < 1.5$, where d_{85} and d_{50} are the cumulative percent passing 85% and 50% respectively. The d_{50} sizes modelled by the various workers typically ranged from about 0.1 mm to 2 or 3 mm.

Probably the most well known early work is that due to Durand and Condolios [6] (in French) who tested a wide range of sands and gravels from 0.2 mm to 25 mm in pipe diameters from 40 mm to 580 mm. They correlated the deposit velocities in terms of the Froude Number (Eqn 1) involving pipe diameter D and the gravitational constant g , which did not allow for solids density. When plotted against particle size, F_L' had an asymptotic value of 1.9 for large particles.

$$V_d = F_L' (gD)^{0.5} \quad (1)$$

Around the same time Worster [45], based on his tests on coarse coal and gravel in 75 mm and 150 mm pipes, produced a similar correlation which did allow for solids density. Subsequently Durand [7] modified the [6], correlation to include solids density and a factor 2,

and plotted the F_L in Eqn 2, versus particle size, with concentration as a parameter. S is the solids density relative to fluid density, ρ_s/ρ_f (both in kg/m^3).

$$V_d = F_L [2 g D (S-1)]^{0.5} \quad (2)$$

Based on the asymptotic $F_L'=1.9$ value for Eqn 1, Eqn 2 should have resulted in an asymptotic $F_L=1.05$. However, as explained by Miedema and Ramsdell [15], Durand [7] divided the vertical axis only by $\sqrt{2}$ meaning that when F_L from Eqn 2 was plotted against particle size the resulting (incorrect) asymptotic value was 1.34. The error was corrected by Condolios and Chapus [3]. Wakefield and Thorvaldsen [33], provided the revised Condolios and Chapus graph, reproduced here in Figure 1. The F_L asymptotic value for large particles is now about 1.0 for volume concentrations (C_v) 10% and 15% reducing to about 0.92 for $C_v=5\%$ and about 0.83 at $C_v=2\%$. However a lot of authors have apparently followed the original English language paper [7] which gave the dashed curves in Figure 1. As a result the error in [7] has been perpetuated over the years, resulting in a predicted deposit velocity about 1.3 times higher than would have been predicted if Eqn 2 had followed correctly from Eqn 1, based on the [6] data.

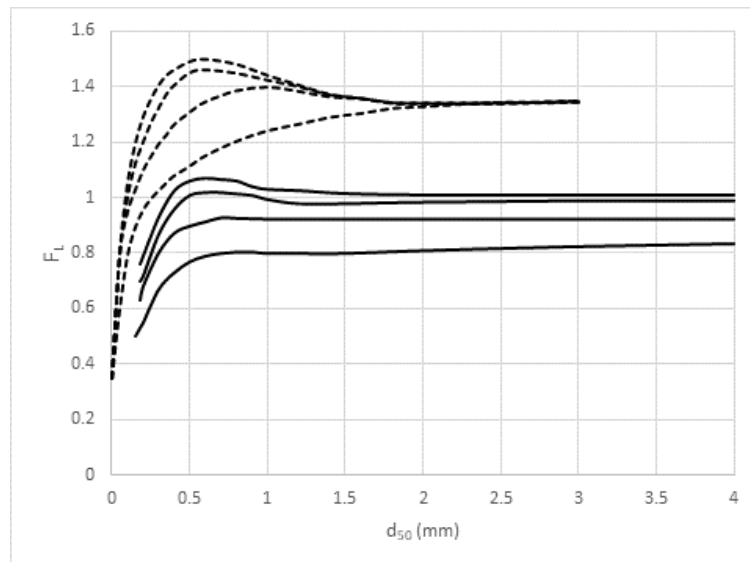


Figure 1. Comparison F_L , Durand [7] (dashed) and corrected graph by [3], each for volume concentrations 2%, 5%, 10% and 15%.

Govier and Aziz [9] provide a comprehensive summary of the many empirical deposit velocity correlations proposed by various authors up until 1972. Note, however, that they, like many others, gave the incorrect F_L from [7] shown dashed in Figure 1.

The full-line [3] curves shown in Figure 1 for d_{50} greater than 0.3 mm are not directly relevant to the current paper. Since the curves end at about $d_{50}=0.2$ mm, only a very small part on the left of Figure 1 is relevant to the current focus on $d_{50}<0.3$ mm slurries.

2.2 Work of Wilson and Co-Workers from 1972 Onwards

Ken Wilson of Queens University, Canada, with co-workers, progressively developed a theory for pipeline flow of discrete solids in water from 1972 onwards. For particles too large to be suspended by turbulence the fundamental basis involved equating a driving force across the ends of a sliding bed with the solid-solid friction force between the bed and the pipe wall,

[35]. A separate theory determined the velocity to initiate turbulent suspension of particles, [36]. Two years later Wilson further developed the theories [37], [38]. The latter authors [38] made this statement: *“In short, it is found that the efficiency of solids pipelines is directly related to the effectiveness of turbulent suspension”*. For $S=2.65$, turbulent suspension only becomes relevant for particles smaller than about 0.3 mm, for which only the extreme left of Figure 1 applies.

Wilson and Judge [39] presented the following equation for the F_L in previous Eqn 2 for medium size particles below about 0.3 mm. Fluid viscosity is included through the particle settling velocity, W .

$$F_L = 2.0 + 0.3 \log_{10} \Delta \quad (3)$$

$$\Delta = 0.75 W^2 / [g D (S-1)] = d/(D C_d) \quad (4)$$

$$C_d = \text{particle drag co-efficient} = 4gd(S-1)/(3W^2) \quad (5)$$

These equations were the basis for the medium particle size (minus 0.3 mm) portion of the Nomogram [40], reproduced below as Figure 2, which applied to silica sand ($S=2.65$) particles, and also the later version incorporating varying solids density, S [41].

Wilson and Judge [39] set a $\Delta > 10^{-5}$ limit on the applicability of Eqn 3, which limits the Nomogram to a minimum particle size of 0.15 mm, as seen in Figure 2. A very important feature of equations (3 & 4) is that they predict a reduction in the dependence of V_d on D as particle size reduces, whereas the Durand type equations (Eqns 1 & 2) show a constant $D^{0.5}$ dependence for all particle sizes. This will be further discussed in Section 2.4.

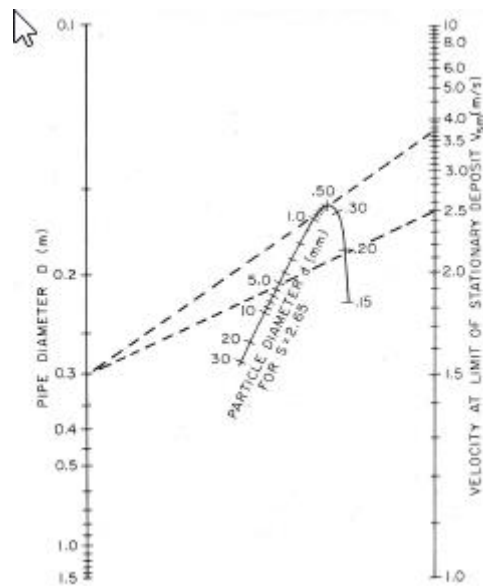


Figure 2. Wilson & Judge [39] Nomogram. V_d for mono-size particles in water, $S=2.65$

2.3 Viscous Sublayer Deposition

The importance of the viscous sublayer in turbulent pipe flow has long been recognised in regard to pressure gradient prediction. Shields [19] also considered the viscous sub-layer for the case of incipient motion of a single particle in open channel flow. It is also important in regard to deposition of very fine particles. For example, Thomas [30] developed a separate

correlation for V_d for particle sizes smaller than the thickness of the laminar sub-layer where the sub-layer thickness was defined as:

$$\delta = 5\mu_f / (\rho_f V^*) \quad (6)$$

where μ_f is the fluid viscosity in Pa.s, V^* is the friction velocity given by $V^* = V\sqrt{f/2}$ and f is the Fanning friction factor.

Thomas [23], applied Wilson's sliding bed theory to particles smaller than the viscous sub-layer and presented the following equation for V_d in terms of friction velocity:

$$V_{d\delta} = 1.1[g \mu_f (S - 1)/\rho_f]^{1/3} \quad (7)$$

If the friction factor is approximated by the Blasius form [13], ($f = 0.046 \text{ Re}^{-0.2}$), where Re is the Reynolds number, then it can be shown that Eqn 7 can be written as:

$$V_{d\delta} = 9.0 [g \mu_f (S-1)/\rho_f]^{0.37} (D \rho_f / \mu_f)^{0.11} \quad (8)$$

Note that Eqns 7 and 8 do not contain particle size. The $d\delta$ subscript identifies it as the viscous sub-layer deposit velocity. To differentiate this from the V_d discussed in Sections 3.2 and 3.3, V_d determined using Eqns 3 and 4 will henceforth be identified as $V_{d\Delta}$.

2.4 From 2000 Onwards – Modification of Wilson & Judge [39] Theory

Thomas [28] modified the, [39] (W&J) equation (Eqn 3) to give the Modified Wilson and Judge (MW&J) equation, Eqn 9.

$$F_L = 2 + 0.305 \log_{10}\Delta + 1.1 \times 10^{-4} \Delta^{-0.489} - 0.044 (1 \times 10^{-7} \Delta)^{-1.06} \quad (9)$$

Figure 3 is taken from [28] but with the addition of the uppermost dashed line showing V_d from Eqn 2 with $F_L=1.0$ as per Condolios and Chappus [3] for C_v between 10% and 15%. The thick grey lines are the W&J predictions using Eqns 3 and 4, and illustrate the limits of applicability of the W&J prediction. The dashed curves beyond these limits show why the limits apply, as the W&J predictions rapidly become meaningless. The full black lines extending from the W&J predictions are the [28] MW&J predictions using Eqn 9. Note the lower slope of the MW&J predictions compared with the 0.5 slope of Eqn 2 as illustrated by the Condolios & Chappus line.

The thick black curve near the bottom of the graph is the viscous sub-layer prediction as per Eqn 7. Note how, as the particle size decreases, the MW&J predicted $V_{d\Delta}$ curves approach the same slope as the viscous sub-layer prediction given by Eqn 7 (or Eqn 8), at least for the industrially meaningful pipe size range 100 mm to 1000 mm.

Regarding the effect of concentration on MW&J predictions, the author's experience, e.g. [23], has been that the maximum V_d for medium to fine sand in water slurries, almost always occurs at a volume concentration around 12%. The wide-ranging data of Schriek et al [18] shows a similar result. Hence the predicted V_d in Figure 3 is assumed to represent the maximum V_d for medium to fine sand in water slurries. The effect of concentration on MW&J predictions for more viscous slurries has not yet been investigated. It is possible that

the effect of concentration on viscous sub-layer deposition studied by Sanders et al [17] may be of some relevance in this regard.

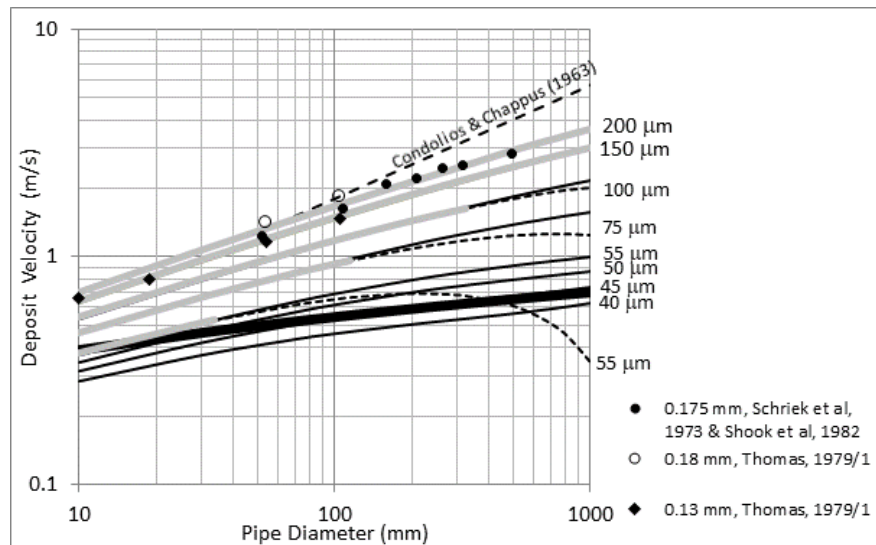


Figure 3. Thomas [28], Modified Wilson & Judge (MW&J) c.f. W&J

2.5 Viscosities Other than Water

The Figures 1, 2 and 3 presented so far, assume silica sand particles in water, although all equations allow for differing solids and fluid densities through S . Also, apart from the Durand type equations (1 and 2), all the other equations also include fluid viscosity and so are applicable to viscosities other than water. The effect of higher fluid viscosities on prediction trends will now be considered.

Sanders et al [17] provide data for a $d_{50}=0.169$ mm sand, in a fluid of density 1170 kg/m^3 and viscosity 6 mPas in a 264 mm ID pipe. MW&J predictions for this viscosity are shown in Figure 4 for d_{50} sizes 0.35 mm , 0.30 mm , 0.25 mm , 0.20 mm , 0.169 mm , 0.15 mm and 0.135 mm , together with the thick line, viscous sub-layer prediction as per Eqn 7. These predictions will be used to illustrate the effect of viscosity on prediction trends by comparing with the predictions for sand in water shown in Figure 3.

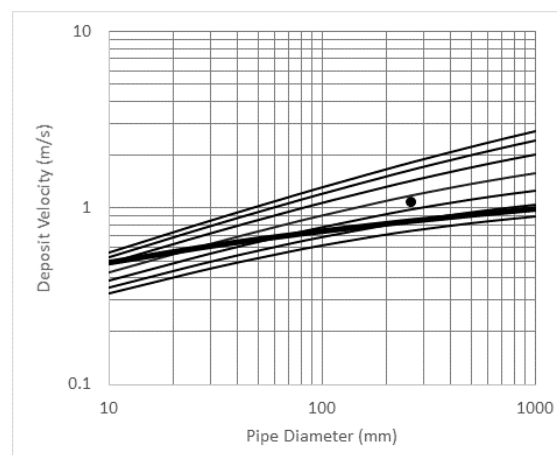


Figure 4 MW&J Predictions for sand in a 6 mPas viscosity fluid, in a 264 mm ID pipe.

The observed V_d was 1.07 m/s as shown by the single data point which is just above the predicted $V_d=0.98$ m/s on the MW&J curve for $d_{50}=0.169$ mm.

Let us now compare Figure 3 and 4 predictions for the largest 1000 mm pipe. The highest predicted curve in Figure 4 is for $d_{50}=0.35$ mm. It predicts $V_d=2.71$ m/s in the 1000 mm pipe. This compares with the highest curve in Figure 3 which applies to a smaller $d_{50}=0.20$ mm, but predicts a higher $V_d=3.63$ m/s for $D=1000$ mm. So, as would be expected, increasing the fluid viscosity from 1 mPas (for water) to 6 mPas and increasing the “fluid” density to 1170 kg/m³, gives a significantly lower predicted MW&J $V_{d\Delta}$ for any given particle size.

The increase in viscosity from 1 mPas to 6 mPas also affects the predicted viscous sub-layer $V_{d\delta}$. Eqn 8 indicates that a 6 fold increase in viscosity *increases* $V_{d\delta}$ by approximately $6^{0.26}$ or 1.59. The contribution due to ρ_f and S reduces this somewhat, with the overall result that (using the full equation 7), the predicted $V_{d\delta}$ for $D=1000$ mm increases from 0.70 m/s for sand in water in Figure 3 to 0.99 m/s in Figure 4.

Thus, an increase in viscosity *reduces* $V_{d\Delta}$ but *increases* $V_{d\delta}$. As a slurry becomes increasingly fine, the deposit velocity is increasingly likely to be determined by viscous sub-layer deposition.

3. GROUP 2 STUDIES, NON-NEWTONIAN SLURRIES

3.1 Homogeneous Non-Newtonian Slurries

Group 2 studies involve the prediction of non-Newtonian laminar and turbulent pipe flow of colloidal type slurries based on the rheology measured under laminar flow conditions, either in a tube viscometer or in a rotational viscometer. Laminar flow of slurries can often be described by the Bingham plastic model [9]. Studies into the turbulent flow of Bingham plastics began in the 1950's with, for example [11], [32] among others. D.G. Thomas was a quite prolific researcher into the flow of Bingham plastics in the 1960's, who recognised that the turbulent flow friction factor falls below the normal Blasius (or Nikuradse) line as transition is approached [31]. Bain and Bonnington [2] also show data illustrating this effect. This type of behaviour had also long been recognised in industrial scale pipe flow, e.g. Wasp et al [34]. Wilson and Thomas [42] developed a theory of Bingham plastic turbulent flow which explains the phenomenon in terms of thickening of the viscous sub-layer.

It should be noted however that Hanks and Dadia [10] developed a theory at variance with this behaviour. Their theory predicts an *increase* in the friction factor above the normal Blasius line as transition is approached. It is possible that they based their theory on experimental data which exhibited this increase, which may have been due to settling of some coarser particles as transition approached.

With regard to the transition velocity between laminar and turbulent flow, V_t . By analogy with Newtonian fluid transition, several authors, including [34], have arrived at the following equation for the transition velocity for a Bingham plastic, where τ_y is Bingham yield stress in Pa.

$$V_t = K (\tau_y/\rho_f)^{0.5} \quad (10)$$

If Newtonian transition is assumed to occur at $Re = 2100$, then $K = 19$. If Newtonian transition is assumed to occur at $Re = 3000$, then $K = 22$. More recently, Slatter and Wasp [21], by correlating a range of data, found that $K = 26$, applicable to $He > 10^5$. Using a theoretical approach based on their 1985 theory of non-Newtonian turbulent flow, Wilson and Thomas [43] determined $K = 25$.

The $He > 10^5$ range covers typical rheology and pipe sizes of interest in slurry pipelining. The significance of Eqn 10 is that, since it does not contain pipe diameter, the transition velocity of any particular Bingham plastic is the same for all pipe sizes and V_t depends only on yield stress and slurry density.

Figure 5 shows results for a kaolin slurry ($\rho_s = 2370 \text{ kg/m}^3$) with slurry density 1100 kg/m^3 , yield stress 7.7 Pa and plastic viscosity $4.9 \text{ mPa}\cdot\text{s}$ in a 18.9 mm and 105 mm pipe [22]. The observed transition velocity is around 2.1 m/s in both pipes, as is predicted by Eqn 10 with the constant either 25 or 26. In contrast, for a Newtonian fluid in these two size pipes, the transition velocity would differ as per the diameter ratio of 5.55. i.e. If as an example, V_t was 2.2 m/s in the 18.9 mm pipe for a Newtonian fluid, then V_t would be only 0.40 m/s in the 105 mm pipe. This would be a large and discernible difference, quite distinct from the Bingham plastic behaviour in Figure 5.

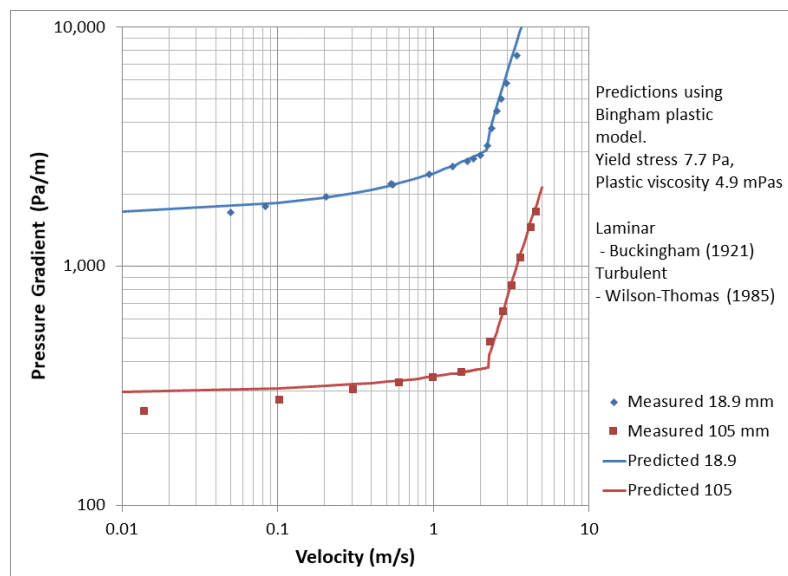


Figure 5 Kaolin clay slurry in 18.9 mm and 105 mm pipes illustrating V_t at same velocity

3.2 Deposition of Non-Newtonian Slurries

Colloidal particles are normally defined as being less than about $1 \mu\text{m}$. Such slurries will flow laminarily without deposition in any size pipe. However, many clay slurries, although predominantly colloidal, do have a proportion of particles larger than $2 \mu\text{m}$. For example, early workers [6] describe slurries with particles up to $40 \mu\text{m}$ as “non-settling”, and Pullum et al [16] found that only particles less than about $40 \mu\text{m}$ contributed to the rheology. This suggests that a slurry composed of particles less than $40 \mu\text{m}$ should also flow laminarily without deposition. This may or may not be the case. It is possible that even $40 \mu\text{m}$ size particles may settle under laminar flow conditions in a pipeline many kilometres long. But once some particles significantly coarser than $40 \mu\text{m}$ are included, it is almost certain that

settling and deposition will eventually occur if the pipeline is long enough, unless the pressure gradient is high enough to slide any settled bed under laminar flow conditions.

The possibility of stable laminar flow with some coarse particles received much attention during the “Stabflo” debate of the late 1970’s and early 1980’s. Thomas [24] first warned of the possibility of settling unless the pressure gradient was high enough. Laminar flow settling has since been studied by numerous authors, including for example, Aude et al [1] and Cooke [4].

It is now known that if the laminar flow pressure gradient is high enough, then stable laminar flow is possible, even with a settled bed of coarse particles present. Cooke [4] gave the critical pressure gradient as between 1 kPa/m to 2 kPa/m for solids with $S=2.65$. Houman and Johnson [12], describe an operating pipeline, 5.5 km long, 330 mm ID, transporting minus 1.5 mm “grits” in fines in laminar flow. They found that a settled bed was kept in motion if the pressure gradient was maintained above 2 kPa/m.

The lack of such a stable laminar flow criterion in 1989, resulted in the economic disaster of the Belovo-Novosibirsk fine coal pipeline in the USSR. This 258 km, 530 mm diameter pipeline was designed to transport minus 0.5 mm coal slurry under laminar flow conditions. As described by Cowper et al [5], the pipeline slowly blocked and was eventually shut down after four years operation. From the above, if the concentration, and hence pressure gradient, had been high enough, settling would not have occurred. Assuming $S=1.4$ for the coal solids then the 2 kPa/m critical pressure gradient required for stable operation with $S=2.65$ solids determined by Houman and Johnson (2002) will translate to a critical pressure gradient $2(1.4-1)/(2.65-1) = 0.48$ kPa/m. From information provided by Cowper et al (2010), the actual maximum pressure gradient provided by the installed pumps was around 0.12 kPa/m, well below the indicated 0.48 kPa/m required for stable laminar flow operation.

The 2 kPa/m pressure gradient required for stable high concentration laminar flow operation (for $S=2.65$), is about an order of magnitude higher than the pressure gradients typically required for low concentration, slurry pipelines operating in turbulent flow. But it is possible to operate a pipeline in laminar flow intermittently [1]. Coarse particles can be allowed to slowly settle along the pipeline (perhaps over a few days) until the pump pressure rises to a critical value at which time the pipeline can be flushed or operated at a lower concentration in turbulent flow to re-suspend the settled particles. Once the pipeline is flushed, laminar flow operation at higher concentration can be resumed.

For example, it was concluded [1] that:

“a long-distance laminar flow pipeline can be successful if sufficient volumetric capacity is built to allow water flushing and significant additional pressure capability is provided in the pump stations to accommodate the rising pressure losses as coarse particles settle out”.

To design such a pipeline requires a method to predict the rate of settling of coarse particles along the length of the pipeline. A settling criterion must include pipeline length but, as of 2020, a definitive settling criterion which includes pipeline length is yet to be developed.

Although, with the above provisos, a laminar flow pipeline can be successful, the safest approach is to assume that flow must be turbulent to ensure no deposition occurs. For these non-Newtonian slurries the deposit velocity V_d will generally coincide with the transition velocity, V_t given by Eqn 10.

4. DEPOSIT VELOCITY PREDICTION FOR WIDE SIZE DISTRIBUTION SLURRIES

4.1 Wide Size Distribution Slurries to be Considered

As noted in the introduction, this paper is focussing on wide size distribution slurries with a maximum particle size of about 1 mm and maximum d_{50} size around 0.3 mm. In Section 2, deposit velocity prediction methods for narrow graded slurries with d_{50} size from about 40 μm to 0.3 mm were discussed. Section 3 discussed non-Newtonian slurries of essentially colloidal size up to 40 μm . In this section, wide size distribution slurries which include colloidal-size particles and particles up to about 1 mm, studied in [27], are considered. The d_{50} size of these wide size distribution slurries is typically in the range 30 μm to 150 μm .

The rheology of these slurries can be measured in a rotational viscometer and, as noted [27], they typically flow pseudo-homogeneously in turbulent flow down to the deposit velocity, without any noticeable heterogeneous upwards hook in pressure gradient prior to deposition. Deposition is due to either settling of the coarser particles as per [28] MW&J, Eqn 9, or by viscous sub-layer deposition [23], Eqn 7. The interaction between these two deposition criteria was discussed for narrow graded slurries with regard to Figures 3 and 4. Those slurries consisted of discrete narrow graded particles in a fluid of known density and viscosity.

But for a slurry with a wide size distribution ranging from colloidal size to perhaps 1 mm, the big question is: What is the “fluid” portion with its relevant viscosity and density, and what is a representative “coarse” particle size? These are questions commented on in this section.

4.2 Pressure Gradient Prediction Methods for Wide Size Distribution Slurries

The slurries considered in Section 2 all have a narrow particle size distribution, typically with d_{85}/d_{50} less than approximately 1.5. However, the majority of industrial slurries have a wider particle size distribution, sometimes ranging from micron size to many mm. There has been considerable research into pressure gradient prediction of wide particle size distribution slurries in water [44], [15]. Wide size distribution slurries in non-Newtonian fluids have also been studied, [14], [16].

Most studies have concentrated on coarse slurries with maximum particle size coarser than the maximum 1 mm size of interest in this present paper. Also, most studies focus on pressure gradient predictions rather than deposit velocity which is the focus of the present paper. Basically, the solids are separated into the “carrier fluid”, heterogeneous load and the fully stratified load. Two, three, four and even five-layer models have been developed to predict the pressure gradient.

Generally, either the minus 40 μm , or minus 75 μm portion is chosen as the “vehicle” or “carrier” portion. As early as 1952, particles less than 40 microns were being classified as being transported as a homogeneous suspension [6]. In 1970, it was stated that “particles less than about 40 μm readily form a homogeneous suspension, even under conditions of laminar flow”, [2]. Pullum et al [16] assume the minus 40 μm portion is the carrier fluid but that this, together with the remaining minus 200 μm particles can be considered part of the pseudo-homogeneous “carrier equivalent fluid”.

For the minus 1 mm slurries of interest in this present paper, the minus 40 μm , or minus 75 μm , vehicle slurry will generally have non-Newtonian properties, most easily modelled as a Bingham plastic.

4.3 Two Approaches for Deposit Velocity Prediction for Wide Size Distribution Slurries

For deposit velocity prediction of the minus 1 mm, viscous slurries of most interest here, two approaches have been taken in recent years. **The first approach**, [27], did not separate the slurry into a “carrier” and a coarse portion, but determined the “inherent viscosity” [25] from the measured rheology of the total slurry, and adopted the weighted mean particle size of the whole slurry as the relevant particle size when predicting deposit velocity. This approach gave predictions consistent with the operational experience in two 593 mm ID gold tailings pipelines [26].

The second approach, for example, Thomas [29], followed the approach generally adopted to predict pressure gradient as described above in Section 4.2, and selected a fine particle portion as the “carrier”. For the minus 300 μm , $d_{50}=40\text{ }\mu\text{m}$, gold tailings of Goosen and Paterson [8], Thomas selected the minus 75 μm portion for the carrier rather than the alternate minus 40 μm , and assumed the median (110 μm) size of the plus 75 μm portion represented the coarse fraction. The predicted trends showed excellent agreement with the measured deposit velocity trends from test loop data over a range of concentrations, for pipe sizes 100 mm, 152 mm and 242 mm [8]. Results for the 100 mm pipe are presented below.

Goosen and Paterson had measured the rheology of the *total* slurry as a Bingham plastic. The yield stress and slurry density of the total slurry was used to predict the transition velocity using eqn 10 with $K=25$. Also, from this total rheology Thomas [29] estimated the rheology of the minus 75 μm carrier following the procedure of Thomas [25]. The rheology of the carrier thus determined, was also a Bingham plastic. For example, for total slurry volume concentration 30%, Goosen and Paterson measured yield stress 1.70 Pa and plastic viscosity 11.4 mPas. The estimated rheology of the minus 75 μm carrier was: Yield stress 1.34 Pa and plastic viscosity 9.01 mPas.

Thomas [29] predicted the deposit velocity at various concentrations using both the plastic viscosity of the carrier and also using the effective viscosity at any particular wall shear stress, based on the turbulent flow wall shear stress predicted using [42]. Figure 6, taken from [29], compares predicted and measured deposit velocity trends in the 100 mm internal diameter test loop of [8].

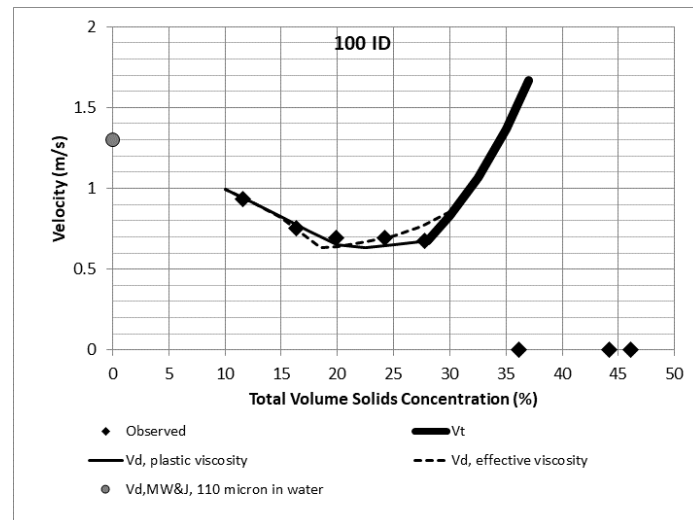


Figure 6 Thomas [29] predictions compared with Goosen & Paterson [8] observations in 100 mm pipe

As previously discussed, there are three types of deposition: Transition, viscous sub-layer, and heterogeneous as per MW&J Eqn 9.

Transition deposition: The thick, full line on the right of Figure 6 represents the predicted transition velocity based on Eqn 10. The three data points for concentrations above 35% are shown at zero m/s indicating no deposition was observed. For the two data points around $C_v=45\%$ the predicted pressure gradients at transition exceed 2.5 kPa/km so it is likely that no deposition would occur at these concentrations under laminar flow conditions, even in a long pipeline. However, at $C_v=36.14\%$, the predicted pressure gradient at transition is only 0.35 kPa/m, well below 2 kPa/m, indicating that laminar flow without deposition could not be sustained, and deposition would have coincided with transition if the test loop had been longer. It can be noted that in the 242 mm pipe loop (not presented here), see [29], deposition was observed close to the transition curve, because settling would have occurred faster because of the much lower pressure gradient in the 242 mm pipe than in the 100 mm pipe.

Viscous sub-layer deposition: At concentrations below where transition is relevant (below 28% to 30%), viscous sub-layer deposition based on Eqn 7 (or Eqn 8) predominates. If the plastic viscosity is used, then viscous sub-layer deposition predominates at concentrations between 20% and 28%. If the effective viscosity is used instead of the plastic viscosity, then the dashed curve in Figure 6 indicates viscous sub-layer deposition predominates from 18% to 30% concentration.

MW&J deposition: At concentrations below 20% (or 18%) the deposit velocity is determined by the MW&J equation 9. As the concentration decreases in Figure 6, the viscosity of the “carrier” portion decreases and so $V_{d\delta}$ decreases and $V_{d\Delta}$ increases and therefore $V_{d\Delta}$ predominates at concentrations below about 20%. Continued reduction in concentration results in a higher deposit velocity as the viscosity (and density) of the carrier fluid continues to decrease.

Comparisons between predicted and observed deposit velocities for the 152 mm and 242 mm test loops show similar trends and agreement [29].

Although the two approaches for deposit velocity prediction outlined above, each gave good results, it is likely that there was a certain amount of good luck in their success. They cannot both be ideal for each case. There is more work to be done in regard to deposit velocity prediction for minus 1 mm viscous slurries.

5. CONCLUSIONS

This paper has focussed on methods of predicting the deposit velocity for wide particle size slurries with maximum particle size up to about 1 mm and maximum d_{50} size around 0.3 mm. These slurries generally possess non-Newtonian properties, typically modelled as Bingham plastics, and typically flow pseudo-homogeneously in turbulent flow down to the deposit velocity. Because they flow pseudo-homogeneously, pressure gradient prediction is relatively easy, and the deposit velocity is the most important parameter as it determines a suitable operating velocity.

Methods of predicting the deposit velocity for mono-size particles in water and in viscous Newtonian fluids are first reviewed, followed by prediction techniques for near-colloidal sized non-Newtonian slurries. These include prediction of the transition velocity between laminar and turbulent flow and the critical pressure gradient required to prevent deposition under laminar flow conditions.

Finally, these prediction techniques are combined to apply to minus 1 mm, wide size distribution, viscous slurries commonly encountered in the mining industry.

6. REFERENCES

- [1] Aude, T.C, Derammelaere, R.H., and Wasp, E.J. (1996), Instability of laminar flow in long distance pipelines, and solutions, Coal Utilization & Fuel Systems 21st Int. Tech. Conf., Clearwater, Florida, USA, March 18-21.
- [2] Bain, A.G. and Bonnington, S.T. (1970), The hydraulic transport of solids by pipeline, Pergamon Press, Oxford, UK.
- [3] Condolios, E and Chapus, E.E., "Solids Pipelines", Chemical Engineering, 24 June, 8 July and 22 July, 1963 (1963).
- [4] Cooke, R. (2002), Laminar flow settling: the potential for unexpected problems, Hydrotransport 15 Conf., Banff, Canada, 3-5th June.
- [5] Cowper, N.T., Sobota, J., and Thomas, A.D. (2010), A technical comparison of coal pipeline options, Hydrotransport 18 Conf., Rio de Janeiro, Brazil, 22-24 Sept.
- [6] Durand, R. and Condolios, E. (1952), Compte Rendue des Deuxieme Journees de l'Hydraulique, Societe Hydrotechnique de France, Paris, June 1952.
- [7] Durand, R. (1953), Basic relationships of the transportation of solids in pipes – experimental research. Proc. 5th Minneapolis Int. Hydr. Conv., Int. Assoc. Hydr. Res. pp 89-103.
- [8] Goosen, P. and Paterson, A. (2014), Trends in stationary deposition velocity with varying slurry concentration covering the turbulent and laminar flow regimes, Hydrotransport 19 Conf., Golden, Colorado, 24-26 Sept.
- [9] Govier, G.W. and Aziz, K. (1972), The flow of complex mixtures in pipes, Robert E. Krieger Publishing Company, Malabar, Florida, USA
- [10] Hanks, R.W. and Dadia, B.H. (1971), Theoretical analysis of the turbulent flow of non-Newtonian slurries in pipes, A.I.Ch.E.J., 17, 3, pp 554-557.

- [11] Hedstrom, B.O.A. (1952), *Ind. Eng. Chem.*, 44, p651
- [12] Houman, J. and Johnson, G. (2002), High density disposal of co-thickened Kimberlite slurry using positive displacement pumps – a case study, *Hydrotransport 15 Conf.*, Banff, Canada, 3-5th June.
- [13] Knudsen, J.G. and Katz, D.L. (1958), *Fluid dynamics and heat transfer*, McGraw-Hill Book Co., New York
- [14] Matousek, V., Penik, V., Pullum, L. and Chryss, A. (2015), Experimental study of bed friction in stratified flow with viscoplastic carrier in pipe, 17th Int. Conf., *Transport & Sedimentation of Solid Particles*, Delft, The Netherlands, 22-25 Sept.+
- [15] Miedema, S.A. and Ramsdell, R.C. (2014), The Delft head loss & limit deposit velocity model, *Hydrotransport 19 Conf.*, Golden, Colorado, USA, 24-26 Sept.
- [16] Pullum, L., Chryss, A., Graham, L., Matousek, V. and Penik, V. (2015), Modelling turbulent transport of solids in non-Newtonian carrier fluids applicable to tailings disposal, 17th Int. Conf. on *Transport & Sedimentation of Solids Particles*, Delft, The Netherlands
- [17] Sanders, R.S., Gillies, R.G., McKibbin, M.J., Litzenberger, C. and Shook, C.A. (2004) Deposition velocities for particles of intermediate size in turbulent flow. *Hydrotransport 16 Conf.*, Santiago, Chile, 26-28 April.
- [18] Schriek, W., Smith, L.G., Haas, D.B. and Husband, W.H.W. (1973), Experimental studies on the transport of two different sands in water in 2, 4, 6, 8, 10 and 12 inch pipelines. *Experimental Studies on Solids Pipelining of Canadian Commodities for The Canadian Transport Commission and The Transportation Development Agency*, E73-21, Saskatchewan Research Council.
- [19] Shields, A. (1936) Application of mechanical similarity and turbulence studies of sediment motion (In German), *Preussische Versuchsanstalt fur Wasserbau und Schiffbau*, Berlin.
- [20] Shook, C.A., Gillies, R., Haas, D.B., Husband, W.H.W, and Small, M. (1982), Flow of coarse and fine sand slurries in pipelines, *J. of Pipelines*, 3, pp 13-21.
- [21] Slatter, P.T. and Wasp, E.J. (2000), The laminar/turbulent transition in large pipes, 10th Int. Conf. on *Transport & Sedimentation of solids particles*, Wroclaw, Poland
- [22] Thomas, A.D. (1978), Coarse particles in a heavy medium – turbulent pressure drop reduction and deposition under laminar flow, *Proc. Hydrotransport 5 Conf.*, Hannover, Germany, 8-11 May.
- [23] Thomas, A.D. (1979/1), Predicting the deposit velocity for horizontal turbulent pipe flow of slurries, *Int. Jnl, Multiphase Flow*, Vol. 5, pp 113-129
- [24] Thomas, A.D. (1979/2). *Pipelining of Coarse Coal as a Stabilized Slurry – Another Viewpoint*, 4th Int. Tech. Conf. on *Slurry Transportation*, Las Vegas, 28-30 March, 1979.
- [25] Thomas, A.D. (2010) Method of determining the inherent viscosity of a slurry and other rheological trends as illustrated by a data bank of over 200 different slurries. *Hydrotransport 18 Conf.*, Rio de Janeiro, Brazil, Sept 22-24
- [26] Thomas, A.D., Hart, S., Parker, B., and Edwards, I. (2010), Newmont's Boddington Gold Mine tailings pipelines. Hydraulic issues and comparisons with operating data, *Hydrotransport 18 Conf.*, Rio de Janeiro, Brazil, 22-24 Sept.
- [27] Thomas, A.D. (2014), Slurries of most interest to the mining industry flow homogeneously and the deposit velocity is the key parameter. *Hydrotransport 19 Conf.*, Golden, Colorado, 24-26 Sept 2014.
- [28] Thomas, A.D. (2015), A modification of the Wilson & Judge deposit velocity equation, extending its applicability to finer particles and larger pipe sizes, 17th Int. Conf. on *Transport & Sedimentation of Solids Particles*, Delft, The Netherlands

- [29] Thomas, A.D. (2019), Deposition of wide size distribution, Bingham plastic slurries in turbulent pipe flow, 19th Int. Conf. on Transport & Sedimentation of Solids Particles, Capetown , South Africa, 24-27 Sept.
- [30] Thomas, D.G.(1962/1), Transport characteristics of suspensions: Part II, Minimum transport velocity for flocculated suspensions in horizontal pipes. A.I.Ch.E.J. 7, 423-430.
- [31] Thomas, D.G.(1962/2), Transport characteristics of suspensions: Part IV, Friction loss of concentrated-flocculated suspensions in turbulent flow. A.I.Ch.E.J. 8, no.2, 266-271.
- [32] Tomita, Y. (1959), Bull.J.S.M.E., 2, pp10-16
- [33] Wakefield, A.W. and Thorvaldsen, G.S. (2002), Comparison of two-layer model with correlative method for prediction of head losses in two large dredging pipelines carrying sand and gravel, Hydrotransport 15 Conf., Banff, Canada, 3-5 June
- [34] Wasp, E.J., Kenny, P. and Gandhi, R.L. (1977), Solid-liquid flow. Slurry pipeline transportation, Trans Tech Publications, Switzerland.
- [35] Wilson, K.C., Streat, M. and Bantin, R.A. (1972), Slip-model correlation of dense two-phase flow, Hydrotransport 2 Conf., Uni of Warwick, Coventry, England, 20-22 Sept.
- [36] Wilson, K.C. (1972), A formula for the velocity to initiate particle suspension in pipeline flow, Hydrotransport 2 Conf., Uni of Warwick, Coventry, England, 20-22 Sept.
- [37] Wilson, K.C. (1974), Co-ordinates for the limit of deposition in pipeline flow, Hydrotransport 3 Conf., Colorado School of Mines, Golden, Colorado, USA, 15-17 May.
- [38] Wilson, K.C. and Watt, W.E. (1974), Influence of particle diameter on the turbulent support of solids in pipeline flow, Hydrotransport 3 Conf., Colorado School of Mines, Golden, Colorado, USA, 15-17 May.
- [39] Wilson, K.C. and Judge, D.G. (1976), New techniques for the scale-up of pilot plant results to coal slurry pipelines. The Int. Symp. On Freight Pipelines, Washington D.C., USA, Dec., Uni. of Penn. Phil, USA.
- [40] Wilson, K.C. and Judge, D.G. (1978), Analytically based nomographic charts for sand water flow. Proc. 5th Int. Conf. on Hydraulic Transport of Solids in Pipes, Hannover, Germany. BHRA Cranfield, U.K. A1/1-12
- [41] Wilson, K.C. (1979), Deposition-limit nomograms for particles of various densities in pipeline flow, Hydrotransport 6, Canterbury, England, Sept 26th- 28th.
- [42] Wilson, K.C. and Thomas, A.D. (1985), A new analysis of the turbulent flow on non-Newtonian fluids, Can. J. Chem. Eng. 63, 539-546
- [43] Wilson, K.C. and Thomas, A.D. (2006), Analytic model of laminar-turbulent transition for Bingham plastics, Can. J.. Chem. Eng., 84, Oct, pp520-526.
- [44] Wilson, K.C., Addie, G.R., Sellgren, A. and Clift, R. (2010), Slurry transport using centrifugal pumps, 3rd Edition, Springer Science+Business Media, Inc.
- [45] Worster, R.C. (1952), The hydraulic transport of solids, Proc. Colloquium on Hydraulic Transport of Coal, National Coal Board, London, Nov.